MAT137 - Integration of Rational Functions

- Today's lecture will assume you have watched videos 9.15

  For Monday's lecture, watch videos 10.1, 10.2

Products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

Hint: You will need

- $\frac{d}{dx}[\tan x] = \ldots$
- $\frac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

Problem: What is the integral when $m = 0$, $n = 1$ and $m = 0$, $n = 3$.

Rational integrals

1. Calculate $\int \frac{1}{x+a}\,dx$
2. Reduce to common denominator $\frac{2}{x} - \frac{3}{x+3}$
3. Calculate $\int \frac{-x+6}{x^2+3x}\,dx$
4. Calculate $\int \frac{1}{x^2+3x}\,dx$
5. Calculate $\int \frac{1}{x^3-x}\,dx$

The integral of secant

Compute

$$\int \sec x \, dx$$

using the substitution $u = \sin x$.

Repeated factors

Do this as an exercise

1. Calculate $\int \frac{1}{(x+1)^n}\,dx$ for $n > 1$
2. Calculate $\int \frac{x}{(x+1)^2}\,dx$
3. Calculate $\int \frac{2x+6}{(x+1)^2}\,dx$
4. Calculate $\int \frac{x^2}{(x+1)^3}\,dx$
5. How would you calculate $\int \frac{\text{polynomial}}{(x+1)^3}\,dx$?