- Today's lecture will assume you have watched videos 9.5, 9.6, 9.10

  For Tuesday's lecture, watch videos 9.15

Computation practice: Integration by parts

Use integration by parts (possibly in combination with other methods) to compute:

1. $\displaystyle\int xe^{-2x}\,dx$
2. $\displaystyle\int x^2 \sin x\,dx$
3. $\displaystyle\int \ln x\,dx$
4. $\displaystyle\int x \arctan x\,dx$
5. $\displaystyle\int \sin\sqrt{x}\,dx$
6. $\displaystyle\int x^2 \arcsin x\,dx$
7. $\displaystyle\int e^{\cos x} \sin^3 x\,dx$
8. $\displaystyle\int e^{ax} \sin(bx)dx$

Exp-trig antiderivative

We want to compute

$$I = \int e^{ax} \sin(bx)\, dx$$

- Try once integration by parts choosing $u = e^{ax}$. Stop.
- Go back to I. Now try integration by parts once choosing $u = \sin(bx)$ instead. Stop.
- Look at what you did. Think.

Persistence

Compute

- $\int_1^e (\ln x)^4 \, dx$
- $\int_1^e (\ln x)^{10} \, dx$

There is a more efficient approach. Call

$$I_n = \int_1^e (\ln x)^n \, dx$$

Use integration by parts on I_n. You will get an equation with I_n and I_{n-1}. Now solve the previous questions.

Practice: Integrals with trigonometric functions

Compute the following antiderivatives. (Once you get them to a form from where it is easy to finish, you may stop.)

1. $\displaystyle\int \sin^{10} x \cos x \; dx$
2. $\displaystyle\int \sin^{10} x \cos^3 x \, dx$
3. $\displaystyle\int e^{\cos x} \cos x \sin^5 x \, dx$
4. $\displaystyle\int \cos^2 x \, dx$
5. $\displaystyle\int \sin^4 x \, dx$
6. $\displaystyle\int \csc x \, dx$

Here are some useful trig identities:

$$\sin^2 x + \cos^2 x = 1 \qquad \sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\tan^2 x + 1 = \sec^2 x \qquad \cos^2 x = \frac{1 + \cos(2x)}{2}$$

A reduction formula

Let $I_n = \int_0^{2\pi} \sin^n x \, dx$.

1. Compute I_0 and I_1.
2. Starting with I_n, use integration by parts.
   Then use the main trig identity to obtain an equation involving I_n and I_{n-2}.
3. Use the previous answers to get a formula for I_n for every positive integer n.
4. Compute I_8. (The answer should be $\frac{35}{64}\pi$).

Products of secant and tangent

To integrate

$$\int \sec^n x \tan^m x \, dx$$

- If $\boxed{???}$, then try the substitution $u = \tan x$.
- If $\boxed{???}$, then try the substitution $u = \sec x$.

Hint: You will need

- $\frac{d}{dx}[\tan x] = \ldots$
- $\frac{d}{dx}[\sec x] = \ldots$
- The trig identity involving sec and tan

Problem: What is the integral when $m = 0$, $n = 1$ and $m = 0$, $n = 3$.