

- Today's lecture will assume you have watched videos 6.1, 6.2

**For Monday's lecture, watch videos 6.4, 6.5, 6.7, 6.8, 6.10**

# Challenging Question

## Theorem

Let  $f$  be differentiable on  $\mathbb{R}$ . Then  $f'$  cannot have jump or removable discontinuities.

Let's first prove this lemma.

## Lemma

Let  $f$  be differentiable on  $\mathbb{R}$ .

IF

- $\lim_{x \rightarrow a^+} f'(x)$  exists,
- $\lim_{x \rightarrow a^-} f'(x)$  exists,

THEN  $\lim_{x \rightarrow a} f'(x) = f'(a)$

## The classic farmer problem

A farmer has  $300m$  of fencing and wants to fence off a rectangular field and add an extra fence that divides the rectangular area in two equal parts down the middle. What is the largest area that the field can have?

You hear a scream. You turn around and you see Ahmed is on fire.

Luckily, you are next to a river. Ahmed is 10 meters away from the river and you are 5 meters away from the point  $P$  on the river closest to Ahmed. You are carrying an empty bucket. You can run twice as fast with an empty bucket as you can run with a full bucket. How far from the point  $P$  should you fill your bucket in order to get to Ahmed with a bucket full of water as fast as possible?