MAT137 - BCT and LCT

- Today's lecture will assume you have watched videos 12.9,12.10

  For Monday's lecture, watch videos 13.2, 13.3, 13.4, 13.5, 13.6, 13.7

Warm up: convergent or ∞?

1. $\int_1^\infty \frac{1}{x^2}\, dx$
2. $\int_1^\infty \frac{1}{\sqrt{x}}\, dx$
3. $\int_1^\infty \frac{1}{x}\, dx$
4. $\int_0^1 \frac{1}{x^2}\, dx$
5. $\int_0^1 \frac{1}{\sqrt{x}}\, dx$
6. $\int_0^1 \frac{1}{x}\, dx$
7. $\int_1^\infty \frac{3}{x^2}\, dx$

True or False?

Let $a \in \mathbb{R}$, and let f and g be continuous functions defined on $[a, \infty)$.

Assume that $\boxed{\forall x \geq a, \quad 0 \leq f(x) \leq g(x)}$.

What can we conclude?

1. IF $\int_a^\infty f(x)\,dx$ is convergent, THEN $\int_a^\infty g(x)\,dx$ is convergent.
2. IF $\int_a^\infty f(x)\,dx = \infty$, THEN $\int_a^\infty g(x)\,dx = \infty$.
3. IF $\int_a^\infty g(x)\,dx$ is convergent, THEN $\int_a^\infty f(x)\,dx$ is convergent.
4. IF $\int_a^\infty g(x)\,dx = \infty$, THEN $\int_a^\infty f(x)\,dx = \infty$.

True or False? – Part 2

Let $a \in \mathbb{R}$, and let f and g be continuous functions defined on $[a, \infty)$.

Assume that $\boxed{\forall x \geq a, \quad f(x) \leq g(x)}$.

What can we conclude?

1. IF $\int_a^\infty f(x)dx$ is convergent, THEN $\int_a^\infty g(x)dx$ is convergent.
2. IF $\int_a^\infty f(x)dx = \infty$, THEN $\int_a^\infty g(x)dx = \infty$.
3. IF $\int_a^\infty g(x)dx$ is convergent, THEN $\int_a^\infty f(x)dx$ is convergent.
4. IF $\int_a^\infty g(x)dx = \infty$, THEN $\int_a^\infty f(x)dx = \infty$.

True or False? – Part 3

Let $a \in \mathbb{R}$, and let f and g be continuous functions defined on $[a, \infty)$.

Assume that $\boxed{\exists M \geq a \text{ such that } \forall x \geq M, \quad 0 \leq f(x) \leq g(x)}$.

What can we conclude?

1. IF $\int_a^\infty f(x)dx$ is convergent, THEN $\int_a^\infty g(x)dx$ is convergent.
2. IF $\int_a^\infty f(x)dx = \infty$, THEN $\int_a^\infty g(x)dx = \infty$.
3. IF $\int_a^\infty g(x)dx$ is convergent, THEN $\int_a^\infty f(x)dx$ is convergent.
4. IF $\int_a^\infty g(x)dx = \infty$, THEN $\int_a^\infty f(x)dx = \infty$.

A simple BCT application

We want to determine whether $\int_1^\infty \frac{1}{x+e^x}\,dx$ is convergent or divergent.

We can try at least two comparisons:

1. Compare $\frac{1}{x}$ and $\frac{1}{x+e^x}$.
2. Compare $\frac{1}{e^x}$ and $\frac{1}{x+e^x}$.

Try both. What can you conclude from each one of them?

What about $\int_0^1 \frac{1}{x+e^x}\,dx$?

BCT calculations

Use the BCT to determine whether each of the following is convergent or divergent

1. $\displaystyle\int_1^\infty \frac{1+\cos^2 x}{x^{2/3}}\,dx$
2. $\displaystyle\int_1^\infty \frac{1+\cos^2 x}{x^{4/3}}\,dx$
3. $\displaystyle\int_0^\infty \frac{\arctan x^2}{1+e^x}\,dx$
4. $\displaystyle\int_0^\infty e^{-x^2}\,dx$
5. $\displaystyle\int_2^\infty \frac{(\ln x)^{10}}{x^2}\,dx$