MAT137 - Week 3 Lecture 2

- **Reminder:** Problem Set 1 is available on the course website, and is due Thursday, September 26 by 11:59pm.
- Today's lecture will assume you have watched videos 2.5 - 2.6
  For Monday's lecture, watch videos 2.7 - 2.11.

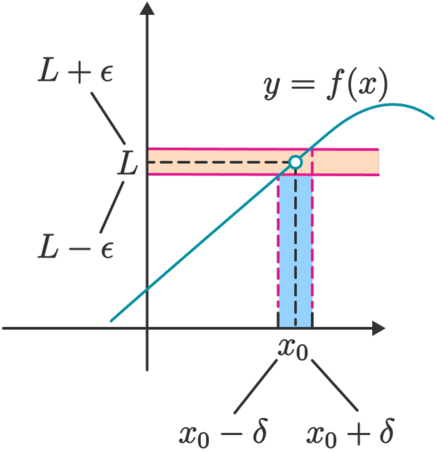
The formal definition of a limit

Write down the formal definition of the statement

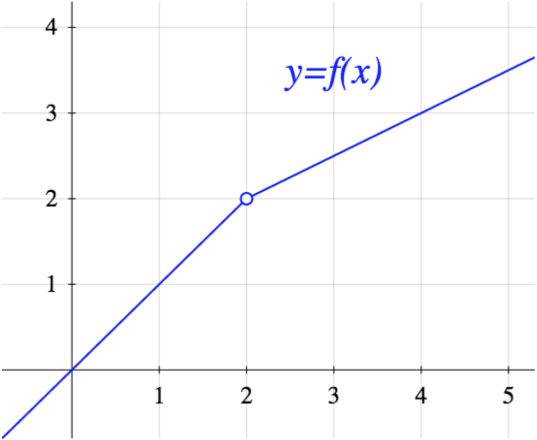
$$\lim_{x \to a} f(x) = L.$$

The formal definition in English

For every interval centred at L, there exists an interval centred at a that is sufficiently small such that f maps this interval inside the interval centred at L.



Understanding the role of δ in the definition



- Find *one* positive value of δ s.t.: $0 < |x-2| < \delta \implies |f(x)-2| < \frac{1}{2}$
- Find *all* positive values of δ s.t.: $0 < |x-2| < \delta \implies |f(x)-2| < \frac{1}{2}$

Defining side limits

Earlier you wrote down the following definition.

Definition

Let $a, L \in \mathbb{R}$. Let f be a function defined at least on an interval centred at a, except possibly at a.

Then

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0 \; \exists \delta > 0 \text{ such that } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Now, write formal definitions of:

$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L$$

Defining infinite limits

In video 2.6, you saw the definition of $\lim_{x \to \infty} f(x) = L$. Let's do a somewhat similar new thing now.

Let $a \in \mathbb{R}$. Let f be a function defined at least on an interval centred at a, except possibly at a.

Write a formal definition for

$$\lim_{x \to a} f(x) = \infty.$$