MAT137 - Improper Integrals and BCT

- Today's lecture will assume you have watched videos 12.1, 12.4, 12.7, 12.8

  For Tuesday's lecture, watch videos 12.9, 12.10

Refining the Big Theorem

1. Construct a sequence $\{x_n\}_{n=0}^{\infty}$ such that

$$n^a \ll x_n \ll n^b \quad \text{for all } a < 2 \text{ and } b \geq 2$$

2. Construct a sequence $\{y_n\}_{n=0}^{\infty}$ such that

$$n^a \ll y_n \ll n^b \quad \text{for all } a \leq 2 \text{ and } b > 2$$

Refining the Big Theorem

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3. Construct a sequence $\{z_n\}_{n=0}^{\infty}$ such that

$$n^a \ll z_n \ll c^n \quad \text{for all } a > 0 \text{ and } c > 1$$

(i.e., construct a sequence that grows much faster than all polynomials, and grows much slower than all exponentials)

Recall the definitions

1. Let f be a bounded, continuous function on $[c, \infty)$.
   How do we define the improper integral
   $$\int_c^\infty f(x)\,dx\,?$$
2. Let f be a continuous function on $(a, b]$.
   How do we define the improper integral
   $$\int_a^b f(x)\,dx\,?$$

Problem 1. Compute the value of this integral, from the definition.

$$\int_1^\infty \frac{1}{x^2+x}dx$$

Hint: $\dfrac{1}{x^2+x} = \dfrac{(x+1)-(x)}{x(x+1)}$.

Problem 1. Compute the value of this integral, from the definition.

$$\int_1^\infty \frac{1}{x^2 + x} dx$$

Hint: $\dfrac{1}{x^2 + x} = \dfrac{(x+1) - (x)}{x(x+1)}$.

Problem 2. Determine whether $\int_0^\infty \cos(x)\, dx$ converges or diverges.

Challenging Problems

Problem 1. Suppose f is continuous on $[1, \infty)$, and that $\lim_{x \to \infty} f(x) = 7$.

What can you conclude about $\int_1^{\infty} f(x)\, dx$? Try to prove your answer.

Challenging Problems

Problem 1. Suppose f is continuous on $[1, \infty)$, and that $\lim_{x \to \infty} f(x) = 7$.

What can you conclude about $\int_1^{\infty} f(x)\,dx$? Try to prove your answer.

Problem 2. Suppose f is continuous on $[1, \infty)$, and suppose you know that $\int_1^{\infty} f(x)\,dx$ converges.

What can you conclude about $\lim_{x \to \infty} f(x)$?

Challenging Problems

Problem 1. Suppose f is continuous on $[1, \infty)$, and that $\lim_{x \to \infty} f(x) = 7$.

What can you conclude about $\int_1^\infty f(x)\,dx$? Try to prove your answer.

Problem 2. Suppose f is continuous on $[1, \infty)$, and suppose you know that $\int_1^\infty f(x)\,dx$ converges.

What can you conclude about $\lim_{x \to \infty} f(x)$?

Suppose you know that $\lim_{x \to \infty} f(x)$ exists, now what can you conclude about $\lim_{x \to \infty} f(x)$?

The most important improper integrals

Use the definition of improper integral to determine for which values of $p \in \mathbb{R}$ each of the following improper integrals converges.

1. $\displaystyle\int_1^\infty \frac{1}{x^p}\,dx$
2. $\displaystyle\int_0^1 \frac{1}{x^p}\,dx.$
3. $\displaystyle\int_0^\infty \frac{1}{x^p}\,dx$