MAT137 - Integration by Substitution

- Today's lecture will assume you have watched videos 9.1,9.4

  For Monday's lecture, watch videos 9.5, 9.6, 9.10

An optimization problem. (Remember those?)

For each real number $a > 0$, consider the function

$$f_a(x) = 1 + a - ax^2.$$

At this point, stop and draw a rough sketch of what the graph of such a function looks like.

Find the value of a that minimizes the area of the region bounded by the graph of f_a and the x-axis.

An application of FTC-1

Use FTC-1 to prove for every $x > 0$ that

$$\int_0^x \frac{dt}{1+t^2} + \int_0^{1/x} \frac{dt}{1+t^2} = \frac{\pi}{2}$$

What's wrong with this computation?

First, note that $-\frac{1}{3x^3}$ is an antiderivative for $\frac{1}{x^4}$.

So we compute:

$$\int_{-1}^{1} \frac{1}{x^4} dx \;=\; -\frac{1}{3x^3}\bigg|_{-1}^{1} = -\frac{2}{3}.$$

However, we know that $\frac{1}{x^4}$ is always positive, and so the definite integral should be a *positive* area.

What's wrong here? Is FTC2 not true?

Substitution exercises.

Problem 1. Consider the following integrals. In each case, what should our u be?

1. $\displaystyle\int e^{7x+5}\,dx$
2. $\displaystyle\int e^{x}\cos\left(e^{x}\right)\,dx$
3. $\displaystyle\int \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx$
4. $\displaystyle\int \frac{x}{1+x^4}\,dx$

Problem 2. Compute $\displaystyle\int \frac{(\ln(\ln x))^2}{x(\ln x)}\,dx$.

Problem 3. Compute $\displaystyle\int x\sqrt[3]{x+3}\,dx$.

A theorem about odd and even functions

Let's use substitution to prove a very useful theorem.

Theorem

Let f be a continuous function defined on all of $\mathbb{R}$.

1. *If f is odd, then for any positive real number a,*

$$\int_{-a}^{a} f(x)\,dx = 0.$$

2. *If f is even, then for any positive real number a,*

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$