MAT137 - Week 10 Lecture 2

- **Reminder:** Problem Set 4 is due Thursday 21 November, by 11:59pm.

- Today's lecture will assume you have watched videos 5.5, 5.6

  For Monday's lecture, watch videos 5.7, 5.8, 5.9, 5.10, 5.11, 5.12

Zeroes of the derivative

Construct a function f that is differentiable on $\mathbb{R}$ and such that

1. f has exactly 2 zeroes and f' has exactly 1 zero.
2. f has exactly 2 zeroes and f' has exactly 2 zeroes.
3. f has exactly 3 zeroes and f' has exactly 1 zero.
4. f has exactly 1 zero and f' has infinitely many zeroes.

How many zeroes?

Let

$$f(x) = e^x - \sin x + x^2 + 10x$$

How many zeroes does f have?

A nice consequence of Rolle's Theorem

The following theorem is missing some of its hypotheses. Fill in the missing hypotheses, then prove the theorem.

Theorem

Let $a < b$ be real numbers. Let f be a function defined on $[a, b]$.

IF

- *(Some conditions about continuity and differentiability.)*
- *f is **not** injective on $[a, b]$*

THEN $\exists c \in (a, b)$ such that $f'(c) = 0$.

1. Write the definition of "f is not injective on $[a, b]$". You will need it.
2. Recall the statement of Rolle's Theorem. You will need that too.
3. Do some rough work to understand why this is true.
4. Write the proof.

The second Theorem of Rolle

Do this as an exercise

Complete statement for this theorem and prove it.

Rolle's Theorem 2

Let $a < b$. Let f be a function defined on $[a.b]$.

IF

- (Some conditions on continuity and derivatives)
- $f(a) = f'(a) = 0$
- $f(b) = 0$

THEN $\exists c \in (a, b)$ such that $f''(c) = 0$.

Hint: Apply the 1st Rolle's Theorem to f, then do something else.