MAT137 - Week 2 Lecture 2

- Course website: http://uoft.me/MAT137
  My page: Course website → Resources → click on my name
  Precalc review: http://uoft.me/precalc
- Office hours: Wednesday 3-5 in PG003
- Today's lecture will assume you have watched videos 1.14 and 1.15.
  For Monday's lecture, watch videos 2.1 to 2.4.

Induction

Suppose we have some statements S_n for all $n \geq 1$.

In each of the following cases, which S_n's will we know are true?

1. **Case 1:** Suppose we have shown that:
   - S_7 is true.
   - $\forall n \geq 1$, S_n is true $\implies S_{n+1}$ is true.
2. **Case 2:** Suppose we have shown that:
   - S_1 is true.
   - $\forall n \geq 7$, S_n is true $\implies S_{n+1}$ is true.
3. **Case 3:** Suppose we have shown that:
   - S_4 is true.
   - $\forall n \geq 1$, S_{n+1} is true $\implies S_n$ is true.
4. **Case 4:** Suppose we have shown that:
   - S_1 is true.
   - $\forall n \geq 1$, S_n is true $\implies S_{n+3}$ is true.

Here is **Case 4** again, from the previous slide:

Suppose we have shown that:

- S_1 is true.
- $\forall n \geq 1$, S_n is true $\implies S_{n+3}$ is true.

What's the least amount of additional work we can do to show that S_n is true for all n?

Induction Exercise

Prove the following by induction.

For any positive integer n, $n^3 - n$ is divisible by 3.

What is wrong with this proof by induction?

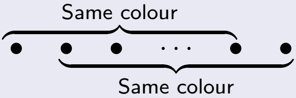
Theorem

$\forall N \in \mathbb{Z}$, in every set of N cars, all the cars are of the same colour.

Proof.

- **Base case.** It is clearly true for $N = 1$.
- **Induction step.**

  Assume it is true for N. I'll show it is true for $N + 1$.
  Take a set of $N + 1$ cars. By induction hypothesis:
  - The first N cars are of the same colour.
  - The last N cars are of the same colour.



Hence the $N + 1$ cars are all of the same colour.

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