MAT137 - Week 2 Lecture 1

- Course website: http://uoft.me/MAT137
  My page: Course website → Resources → click on my name
  Precalc review: http://uoft.me/precalc

- Office hours: Wednesday 3-5 in PG003

- Today's lecture will assume you have watched videos 1.7 through 1.13.
  For Tuesday's lecture, watch videos 1.14 and 1.15.

1. Lectures (from all 7 instructors)
2. practice questions from the book
3. playlist practice questions (obtained here)
4. tutorial worksheet (obtained here).
5. problem set questions

Warm up on conditionals

Four cards lie on the table in front of you. You know that each card has a letter on one side and a number on the other. At the moment, you can read the symbols E, P, 3, and 8 on the sides that are up. I tell you:

> *"If a card has a vowel on one side,*
> *then it has an odd number on the other side."*

Which cards do you need to turn over in order to verify whether I am telling the truth or not?

More on Conditionals

Theorem

Let $n, m \in \mathbb{Z}$.
IF n is not a multiple of 5 and m is not a multiple of 5
THEN $n + m$ is not a multiple of 5.

- Rewrite this theorem without using “not”. (consider the contrapositive).
- Write the negation of the theorem.
- Is the theorem true or false? Prove it.

Definitions - Injectivity

Let f be a function with domain D.

f is called <u>injective on D</u> (or sometimes <u>one-to-one on D</u>) if different inputs to the function always yield different outputs. In other words, different values of x must produce different values of $f(x)$.

For example, the function $f(x) = x$ is injective, while the function $g(x) = x^2$ is not.

Problem: Write a formal definition for this property.

Definitions - Injectivity (continued)

Let f be a function with domain D. Which of these is a definition of "f is injective on D"? For those that are not, what (if anything) *do* they mean?

1. $f(x_1) \neq f(x_2)$.
2. $\exists x_1, x_2 \in D$ such that $f(x_1) \neq f(x_2)$.
3. $\forall x_1, x_2 \in D$, $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$.
4. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$.
5. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.
6. $\forall x_1, x_2 \in D$, $f(x_1) \neq f(x_2) \implies x_1 \neq x_2$.
7. $\forall x_1, x_2 \in D$, $f(x_1) = f(x_2) \implies x_1 = x_2$.
8. $\forall x_1, x_2 \in D$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Definitions - Injectivity (continued)

Let f be a function with domain D. Which of these is a definition of "f is injective on D"? For those that are not, what (if anything) *do* they mean?

1. $f(x_1) \neq f(x_2)$. $\leftarrow$ meaningless.
2. $\exists x_1, x_2 \in D$ such that $f(x_1) \neq f(x_2)$. $\leftarrow$ definition of "f is not constant on D".
3. $\forall x_1, x_2 \in D$, $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$. $\leftarrow$ false (unless $D = \phi$)
4. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$. $\leftarrow$ same as 2. "$x_1 \neq x_2$" is unnecessary
5. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$. $\leftarrow$ Always satisfied; just pick $x_1 = x_2$.
6. $\forall x_1, x_2 \in D$, $f(x_1) \neq f(x_2) \implies x_1 \neq x_2$. $\leftarrow$ definition of "f is a function".

Definitions - Injectivity (continued)

Let f be a function with domain D. Which of these is a definition of "f is injective on D"? For those that are not, what (if anything) *do* they mean?

1. $f(x_1) \neq f(x_2)$.
2. $\exists x_1, x_2 \in D$ such that $f(x_1) \neq f(x_2)$.
3. $\forall x_1, x_2 \in D$, $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$.
4. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2$ and $f(x_1) \neq f(x_2)$.
5. $\exists x_1, x_2 \in D$ such that $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.
6. $\forall x_1, x_2 \in D$, $f(x_1) \neq f(x_2) \implies x_1 \neq x_2$.
7. $\forall x_1, x_2 \in D$, $f(x_1) = f(x_2) \implies x_1 = x_2$.
8. $\forall x_1, x_2 \in D$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Proving that a function is injective

Definition

Let f be a function with domain D. f is injective on D if either of the following equivalent statements is true.

- $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2.$
- $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2).$

Suppose I gave you a specific function f and asked you to prove it is injective.

What would the structure of your proof be if you use each of the equivalent conditions above? How would your proof begin? What would you assume? What would your conclusion have to be?

Problem: Let f be the function defined by $f(x) = 7x + 1$. Prove that f is injective on $\mathbb{R}$.

Proving that a function is NOT injective

Definition

Let f be a function with domain D. f is injective on D if either of the following equivalent statements is true.

- $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$.
- $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$.

Now suppose I gave you a specific function f and asked you to prove it's not injective. You would need to show that f satisfies the negation of the definiton above.

1. Write the negations of both conditions above.
2. What would the structure of your proof be?

Problem: Let f be the function defined by $f(x) = x^2$. Prove that f is not injective on $\mathbb{R}$.