- Today's lecture will assume you have watched videos 4.3,4.4 4.5

  For Monday's lecture, watch videos 4.6, 4.7, 4.8, 5.1, 5.2, 5.3, 5.4

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Find an equation for $h^{-1}(x)$.
3. Sketch the graphs of h and h^{-1}.
4. Verify that for every $t \in \boxed{???}$, $h(h^{-1}(t)) = t$, and that for every $t \in \boxed{???}$, $h^{-1}(h(t)) = t$.

Warm-up. Did you watch the videos?

Problem 1. Let f be a function with domain D. Write the definition of

$$f \text{ is injective on } D.$$

Problem 2. What is the relationship between injectivity and the existence of inverses?

Left and right inverses

Let $f : A \to B$

Problem 1: Which of the following is a sufficient condition for the existence of an inverse?

1. There exists a function $g : B \to A$ such that $\forall x \in A,\ g(f(x)) = x$ (We call g a left inverse)

2. There exists a function $g : B \to A$ such that $\forall y \in B,\ f(g(y)) = y$ (We call g a right inverse)

3. There exists a function $g : B \to A$ such that
    - $\forall x \in A,\ g(f(x)) = x$
    - $\forall y \in B,\ f(g(y)) = y$

Problem 2: In each case, what extra assumption do we need on f so that f will have an inverse?

Composition of injective functions - part one

Assume that all functions in this problem have domain $\mathbb{R}$.

Prove the following theorem:

Theorem

Let f and g be functions.

IF f and g are injective, THEN $f \circ g$ is injective.

How to proceed:

1. Write the definition of what you want to prove.
2. Figure out the structure of the proof.
3. Complete the proof, making sure you have used both hypotheses.

Composition of injective functions - part two

Do this as an exercise.

Assume that all functions in this problem have domain $\mathbb{R}$.

Prove the following theorem:

Theorem

Let f and g be functions.

IF $f \circ g$ is injective, THEN g is injective.

How to proceed:

1. Write the definition of what you want to prove.
2. Figure out the structure of the proof.
3. Complete the proof, making sure you have used both hypotheses.