- Today's lecture will assume you have watched videos 3.19, 3.20, 4.1, 4.2.
  For Tuesday's lecture, watch videos 4.3,4.4 4.5

Related rates

Idea of these problems: If you know a relationship between two quantities, you can derive a relationship between rates of change of those two quantities.

Question: If the radius R of a circle is changing at a rate of 2 m/s, how fast is the area A of the circle changing?

If you know how the area A of a circle relates to its radius R ($A = \pi R^2$), and you know the radius is changing at some rate $\frac{dR}{dt}$, then you can figure out the rate $\frac{dA}{dt}$ at which the area must be changing.

You can do this by differentiating both sides of our relationship with respect to time:

$$A = \pi R^2 \implies \frac{d}{dt}[A] = \frac{d}{dt}\left[\pi R^2\right] \implies \frac{dA}{dt} = 2\pi R \frac{dR}{dt}$$

Classic Related rates problems

Here are two classic related rates problem, to warm us up.

Problem 1. A 10 foot ladder leans against a wall. The bottom of the ladder starts slipping away at a rate of 0.5 feet per second. How quickly is the top of the ladder dropping when the bottom is 4 feet from the wall?

Problem 2. A spherical balloon is being inflated with 1 cubic metre of air per hour. How quickly is its diameter increasing when it is 2 metres in diameter?

Math party

The MAT137 TAs wanted to rent a disco ball for their upcoming party. However, since they are poor, they could only afford a flashlight. At the party, one TA is designated the "human disco ball". The TA stands in the center of the room pointing the flashlight horizontally and spins at 3 revolutions per second. (Yes, they are that fast. Ask your TA to demonstrate on your next tutorial if you don't believe me!) The room is square with side length 8 meters. At which speed is the light from the flashlight moving across the wall when it is 2 meters away from a corner?

Sleepy ants

Two ants are taking a nap. The first one is resting at the tip of the minute hand of a cuckoo clock, which is 25 cm long. The second one is resting at the tip of the hour hand, which is half the length. At what rate is the distance between the two ants changing at 3:30?

Water tanks

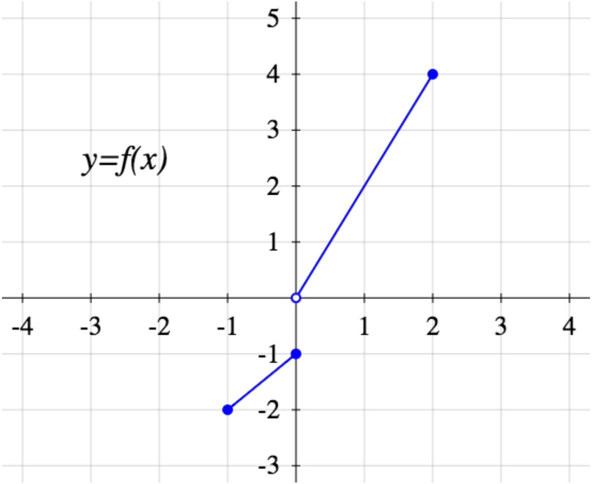
I am posting this for practice but we did not cover it in lecture

A water tank has the shape of a cylinder with radius 1 m and length 2 m. If water is being pumped into the tank at a rate of $\frac{1}{6}m^3$ per minute, find the rate at which the water level is rising when the water is $\frac{1}{2}m$ deep.

Suppose that the cylinder is:

1. vertical
2. horizontal

Inverses from a graph

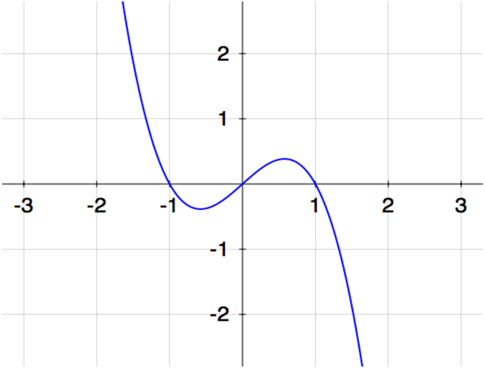


Compute:

1. $f(2)$
2. $f(0)$
3. $f^{-1}(2)$
4. $f^{-1}(0)$
5. $f^{-1}(-1)$

Inverses

Let f be the following function:



1. What is the largest interval containing -1 on which f has an inverse?
2. What is the largest interval containing 0 on which f has an inverse?

Try to sketch the graphs of these two inverses.

Let

$$h(x) = x|x| + 1$$

1. Calculate $h^{-1}(-8)$.
2. Find an equation for $h^{-1}(x)$.
3. Sketch the graphs of h and h^{-1}.
4. Verify that for every $x \in \boxed{???}$, $h(h^{-1}(t)) = t$, and that for every $x \in \boxed{???}$, $h^{-1}(h(t)) = t$.

Left and right inverses

Attempt this question for next lecture

Let $f : A \to B$
Which of the following is a sufficient condition for the existence of an inverse?

1. There exists a function $g : B \to A$ such that $\forall x \in A$, $g(f(x)) = x$
2. There exists a function $g : B \to A$ such that $\forall y \in B$, $f(g(y)) = y$

In each case, what extra assumption do we need on f so that f will have an inverse?