MAT137 - The Big Theorem

- Today's lecture will assume you have watched videos 11.7, 11.8

  For Monday's lecture, watch videos 12.1, 12.4, 12.7, 12.8

Calculations

1. $$\lim_{n\to\infty} \frac{n! + 2e^n}{3n! + 4e^n}$$

2. $$\lim_{n\to\infty} \frac{2^n + (2n)^2}{2^{n+1} + n^2}$$

3. $$\lim_{n\to\infty} \frac{5n^5 + 5^n + 5n!}{n^n}$$

Big Theorem: TRUE or FALSE

Let a_n and b_n be positive sequences.

1. IF $a_n << b_n$ THEN $\forall m \in \mathbb{N},\ a_m < b_m$
2. IF $a_n << b_n$ THEN $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$
3. IF $a_n << b_n$
   THEN $\exists n_0 \in \mathbb{N}$ s.t. $\forall m \in \mathbb{N},\ m \geq n_0 \Rightarrow a_m < b_m$
4. IF $\forall m \in \mathbb{N},\ a_m < b_m$ THEN $a_n << b_n$
5. IF $\exists m \in \mathbb{N}$ s.t. $a_m < b_m$ THEN $a_n << b_n$
6. IF $\exists n_0 \in \mathbb{N}$ s.t. $\forall m \in \mathbb{N},\ m \geq n_0 \Rightarrow a_m < b_m$
   THEN $a_n << b_n$
7. IF $\forall \varepsilon > 0,\ a_n < \varepsilon b_n$ for large enough n,
   THEN $a_n << b_n$

An application of the previous theorem

Prove that

$$\sqrt{2+\sqrt{2+\sqrt{2+\dots}}} = 2$$

1. Explicitly write the expression as the limit of a sequence $\{a_n\}_{n\in\mathbb{N}}$
2. Show that $0 \leq a_n \leq 2$ for all $n \in \mathbb{N}$.
3. Show that $\{a_n\}_{n\in\mathbb{N}}$ is increasing.
4. Conclude that $a := \lim_{n\to\infty} a_n$ exists.
5. Use that $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} a_{n+1}$ and that $f(x) := \sqrt{2+x}$ is continuous at a to solve for a.