MAT137 - Alternating series, Conditional and absolute convergence

- Today's lecture will assume you have watched videos 13.15

  For Monday's lecture, watch videos 13.18, 13.19, 14.1, 14.2

Convergent or divergent?

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sin n}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sin n}$
5. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$
6. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$

An AST example

Verify carefully the 3 hypotheses of the Alternating Series Test for

$$\sum_{n=0}^{\infty} (-1)^n \frac{n-\pi}{e^n}$$

Can we conclude it is convergent?

A counterexample to the AST?

Try to construct a series of the form $\sum_{n=1}^{\infty}(-1)^n b_n$ such that

- $b_n > 0$ for all $n \geq 1$,
- $\lim_{n\to\infty} b_n = 0$,
- the series $\sum_{n=1}^{\infty}(-1)^n b_n$ is divergent.

Hint: First, think about which hypothesis of the Alternating Series Test must fail in order for this to be possible.

A problem from a past final exam

Suppose we know:

- $\forall n \in \mathbb{N}$, $0 < a_n < 1$;
- the series $\sum_{n}^{\infty} a_n$ is convergent,

Determine whether the following series converge, diverge, or we do not have enough information to decide:

1. $\sum_{n}^{\infty} \sin a_n$
2. $\sum_{n}^{\infty} \cos a_n$
3. $\sum_{n}^{\infty} (a_n)^2$
4. $\sum_{n}^{\infty} \sqrt{a_n}$

True or False – Absolute Values

1. IF $\{a_n\}_{n=1}^{\infty}$ is convergent, THEN $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent.
2. IF $\{\,|a_n|\,\}_{n=1}^{\infty}$ is convergent, THEN $\{a_n\}_{n=1}^{\infty}$ is convergent.
3. IF $\sum_{n=1}^{\infty} a_n$ is convergent, THEN $\sum_{n=1}^{\infty} |a_n|$ is convergent.
4. IF $\sum_{n=1}^{\infty} |a_n|$ is convergent, THEN $\sum_{n=1}^{\infty} a_n$ is convergent.

Positive and negative terms – part 1

- Let $\sum a_n$ be a series.
- Call $\sum$ P.T. the sum of only the positive terms of the same series.
- Call $\sum$ N.T. the sum of only the negative terms of the same series.

IF $\sum$ P.T. is...	AND $\sum$ N.T. is...	THEN $\sum a_n$ may be...
CONV	CONV	
∞	CONV	
CONV	$-\infty$	
∞	$-\infty$	