Welcome to MAT137!

(Section L5101, M5-7 in MP102 and T5 in MP202)

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Objectives of the course

- Calculus Concepts
- Mathematical Rigour
- Problem Solving

Important Information

- Course website: http://uoft.me/MAT137
- Make sure you have read and understood the course outline.
  (To find it, go to: Course website $\rightarrow$ Resources.)
- Make sure you check your UofT email regularly for announcements
- Join Piazza, our online help forum. (Seriously, it's great)
  (For links, go to: Course website $\rightarrow$ Resources.)
- Precalculus review: http://uoft.me/precalc
  (Strong precalc skills are the most important prerequisite of this course.)

  Going through material this is "Problem Set 0".

How the Lectures will Run

- You will watch videos **before** the lecture
- During the lecture, we will work on problems and do exercises based on the content covered in the videos. **Participate in class**
- L5101-webpage:
  My page for just our section.
  (To find it, go to: Course website → Resources and click on my name.)

  Please get into the habit of checking both the course website and the page above regularly.

  Our section's page will tell you which videos to watch before each lecture. **For tomorrow's lecture, watch videos 1.7 through 1.9.**

Resources available for you

- Learning the content
  1. Videos
  2. Book
- Practicing
  1. Lectures
  2. Problem set questions
  3. practice questions from the book
  4. playlist practice questions (obtained here)
  5. tutorials
- Asking for help
  1. Office hours
  2. Piazza
  3. MLC (math learning centre)
  4. Proof Hub

Sets 1

Describe the following sets in the simplest terms you can.

1. $[2, 4] \cup (3, 10)$
2. $[2, 4] \cap (3, 10)$
3. $(\pi, 3)$
4. $[7, 7]$
5. $(7, 7)$
6. $A = \{ x \in \mathbb{R} : x^2 < 7 \}$
7. $B = \{ x \in \mathbb{Z} : x^2 < 7 \}$
8. $C = \{ x \in \mathbb{N} : x^2 < 7 \}$

Even numbers

Which of these is a correct description of the set E of even integers?

1. $E = \{n \in \mathbb{Z} : \forall a \in \mathbb{Z}, n = 2a\}$
2. $E = \{n \in \mathbb{Z} : \exists a \in \mathbb{Z} \text{ s.t. } n = 2a\}$

Which of these statements is true?

3. $\forall a \in \mathbb{Z}$, the number $n = 2a$ is even.
4. $\exists a \in \mathbb{Z}$ s.t. the number $n = 2a$ is even.

Sets defined with quantifiers

I am posting this for extra practice but we didn't cover it in lecture

Problem 1. Describe the following sets in the simplest terms you can.

1. $A = \{ x \in \mathbb{R} : \forall y \in [5,7], x < y \}$.
2. $B = \{ x \in \mathbb{R} : \exists y \in [5,7] \text{ such that } x < y \}$
3. $C = \{ x \in [5,7] : \forall y \in [5,7], x < y \}$.
4. $D = \{ x \in [5,7] : \exists y \in [5,7] \text{ such that } x < y \}$
5. $E = \{ x \in [5,7] : \exists y \in \mathbb{R} \text{ such that } x < y \}$
6. $F = \{ x \in [5,7] : y \in \mathbb{R}, x < y \}$

Problem 2. Write a definition of $\mathbb{Q}$ (the set of rational numbers) using set-builder notation.

Some new notation for sets

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Given two sets A and B, we define:

- $A \setminus B = \{ x \in A : x \notin B \}$.

  We usually read this as "A without B" or similar. It's the set consisting of all elements of A that are not elements of B.

- $A \triangle B = (A \setminus B) \cup (B \setminus A)$.

  We usually read this as "the symmetric difference between A and B". It's the set of all elements A or B but not both.

To check your understanding of notation, convince yourself that

$$A \triangle B = (A \cup B) \setminus (A \cap B).$$

Using the notation we just defined

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Problem Define the following two sets:

- $A = \{18 \text{ year old students in this class}\}$
- $B = \{\text{students sitting in the first two rows of this class}\}$

What are the sets $A \setminus B$, $B \setminus A$, and $A \triangle B$?

Sets defined with quantifiers

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Problem: Recall that a real number that is not rational is called *irrational*. For example, π, e, and $-\sqrt{2}$ are all irrational numbers.
Let A be the set of all negative, rational numbers and positive, irrational numbers.

Write a definition of A using mathematical notation. (There is more than one way to do this. Feel free to use the words "and", "or", etc.)

Write A in terms of $\mathbb{Q}$ and the set of positive real numbers $\mathbb{R}_{>0}$

Domination

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(This is a problem from a previous year's first problem set.)

If A and B are both sets of real numbers, we say B *dominates* A if the following is true:

For every $a \in A$, there exists $b \in B$ such that $a < b$.

If you prefer mathematical notation:

$$\forall a \in A, \exists b \in B, \text{ such that } a < b.$$

Problem. Find two non-empty sets of real numbers A and B such that the following three things are true:

1. $A \cap B = \emptyset$.
2. A dominates B.
3. B dominates A.

Indecisive function

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Construct a function f that satisfies all of the following properties at once:

- The domain of f is $\mathbb{R}$.
- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that

$$x < y \text{ and } f(x) < f(y)$$

- $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ such that

$$x < y \text{ and } f(x) > f(y)$$