MAT137 - Integral and Comparison tests, Alternating series

- Today's lecture will assume you have watched videos 13.10, 13.11, 13.12, 13.13, 13.14

  For Tuesday's lecture, watch videos 13.15

Rapid fire

For which values of $a \in \mathbb{R}$ are these series convergent?

1. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{a^n}$
2. $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^a}$
3. $\displaystyle\sum_{n=0}^{\infty} a^n$
4. $\displaystyle\sum_{n=0}^{\infty} n^a$

More rapid fire

Convergent or divergent?

1. $$\sum_{n}^{\infty} \frac{n^{10} + 17n^7 + 3}{n^{11}}$$
2. $$\sum_{n}^{\infty} \frac{\sqrt[3]{n^2+1} + 1}{\sqrt{n^4+n} + n + 1}$$
3. $$\sum_{n}^{\infty} n! e^{-n}$$
4. $$\sum_{n}^{\infty} \frac{3^n}{n 2^{2n}}$$

Necessary condition: TRUE or FALSE

1. IF $\lim_{n\to\infty} a_n = 0$, THEN $\sum_n^{\infty} a_n$ is convergent.
2. IF $\lim_{n\to\infty} a_n \neq 0$, THEN $\sum_n^{\infty} a_n$ is divergent.
3. IF $\sum_n^{\infty} a_n$ is convergent THEN $\lim_{n\to\infty} a_n = 0$.
4. IF $\sum_n^{\infty} a_n$ is divergent THEN $\lim_{n\to\infty} a_n \neq 0$.

More TRUE or FALSE

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic,
   THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.
2. IF the series $\sum_{n=0}^{\infty} a_n$ converges
   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.
3. If $\forall n \in \mathbb{N}, a_n > 0$ THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing
4. If $\sum_{n=0}^{\infty} a_n$ is convergent THEN $\lim_{k\to\infty} \left[\sum_{n=k}^{\infty} a_n \right] = 0$

Even more TRUE or FALSE

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence. s

1. IF $\lim_{n\to\infty} S_{2n}$ exists, THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
2. IF $\lim_{n\to\infty} S_{2n}$ exists and $\lim_{n\to\infty} a_n = 0$,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
3. IF $\sum_{n=0}^{\infty} a_{2n}$ is convergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

   THEN $\sum_{n=0}^{\infty} a_n$ is convergent.
4. IF $\sum_{n=0}^{\infty} a_{2n}$ is divergent and $\sum_{n=0}^{\infty} a_{2n+1}$ is convergent,

   THEN $\sum_{n=0}^{\infty} a_n$ is divergent.

Slower questions

Convergent or divergent?

1. $$\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$$
2. $$\sum_{n}^{\infty} \frac{1}{n \ln n}$$
3. $$\sum_{n}^{\infty} e^{-n^2}$$

Slower questions

Convergent or divergent?

1. $$\sum_{n}^{\infty} \frac{2^n - 40}{3^n - 20}$$
2. $$\sum_{n}^{\infty} \frac{1}{n \ln n}$$
3. $$\sum_{n}^{\infty} e^{-n^2}$$
4. $$\sum_{n}^{\infty} \sin^2 \frac{1}{n}$$
5. $$\sum_{n}^{\infty} \frac{1}{n(\ln n)^3}$$
6. $$\sum_{n}^{\infty} \frac{(\ln n)^{20}}{n^2}$$