MAT137 - Week 5 Lecture 2

- **Reminder:** Problem Set 2 is available on the course website, and is due Thursday, 10 October by 11:59pm.
- Today's lecture will assume you have watched videos 2.19 - 2.20
For Tuesday's lecture, watch videos 2.21 - 2.22.

True or False

- If $f(x) > g(x)$ on an interval centered at a but maybe not a, then $\lim_{x\to a} f(x) > \lim_{x\to a} g(x)$ (assuming the limits exist).
- $\lim_{x\to 0} f(x) = 0$ iff $\lim_{x\to 0} f(f(x)) = 0$
- If g is a non-zero function that is bounded on $\mathbb{R}$, then $\lim_{x\to 0} \frac{g(x)}{x^2} = \infty$
- If g is a function satisfying $|g(x)| > 1$ on $\mathbb{R}$, then $\lim_{x\to 0} \frac{g(x)}{x^2} = \infty$
- If $\lim_{x\to 0} \frac{g(x)}{x}$ exists, then $\lim_{x\to 0} g(x) = 0$

Trigonometric limits

Using that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ (and maybe some simple trig identities), compute the following limits:

1. $\lim_{x \to 0} \frac{\cos x}{x}$
2. $\lim_{x \to 0} \frac{\sin(5x)}{x}$
3. $\lim_{x \to \infty} x \sin \frac{1}{x}$
4. $\lim_{x \to 0} \frac{\sin e^x}{e^x}$
5. $\lim_{x \to 0} \frac{1 - \cos x}{x}$
6. $\lim_{x \to 0} \frac{\sin x}{x^2}$

Compute the following limits.

1. $\lim_{x\to\infty} (x^7 - 2x^5 + 11)$
2. $\lim_{x\to\infty} \left(x^2 - \sqrt{x^5 + 1}\right)$
3. $\lim_{x\to\infty} \frac{x^2 + 11}{x + 1}$
4. $\lim_{x\to\infty} \frac{x^2 + 2x + 3}{3x^2 + 4x + 5}$
5. $\lim_{x\to\infty} \frac{x^3 + \sqrt{2x^6 + 1}}{2x^3 + \sqrt{x^5 + 1}}$

repeat all of them when ∞ is replaced by $-\infty$

Which solution is correct?

Compute $L = \lim_{x\to-\infty}\left[x - \sqrt{x^2+x}\right]$.

- **Solution 1**

$$L = \lim_{x\to-\infty} \frac{\left[x - \sqrt{x^2+x}\right]\left[x + \sqrt{x^2+x}\right]}{\left[x + \sqrt{x^2+x}\right]} = \lim_{x\to-\infty} \frac{x^2 - (x^2+x)}{\left[x + \sqrt{x^2+x}\right]}$$

$$= \lim_{x\to-\infty} \frac{-x}{x\left[1 + \sqrt{1+\frac{1}{x}}\right]} = \lim_{x\to-\infty} \frac{-1}{\left[1 + \sqrt{1+\frac{1}{x}}\right]} = \frac{-1}{2}$$

- **Solution 2**

$$L = \lim_{x\to-\infty}\left[x - \sqrt{x^2+x}\right] = (-\infty) - \infty = -\infty$$