MAT137 - Sequences

- *Reminder:* Test 3 is this Thursday.
  - See the course website for info about Test 3, including what material it will cover.

- Today's lecture will assume you have watched videos 11.1-11.2

  For Monday's lecture, watch videos 11.3, 11.4, 11.5, 11.6

Warm up

Write a formula for the general term of these sequences

1. $\{a_n\}_{n=0}^{\infty} = \{1, 4, 9, 16, 25, \dots\}$
2. $\{b_n\}_{n=1}^{\infty} = \{1, -2, 4, -8, 16, -32, \dots\}$
3. $\{c_n\}_{n=1}^{\infty} = \left\{\frac{2}{1!}, \frac{3}{2!}, \frac{4}{3!}, \frac{5}{4!}, \dots\right\}$
4. $\{d_n\}_{n=1}^{\infty} = \{1, 4, 7, 10, 13, \dots\}$

True or False?

Let f be a function with domain at least $[1, \infty)$.
We define a sequence as $a_n = f(n)$.
Let $L \in \mathbb{R}$.

1. IF $\lim_{x \to \infty} f(x) = L$, THEN $\lim_{n \to \infty} a_n = L$.

2. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{x \to \infty} f(x) = L$.

3. IF $\lim_{n \to \infty} a_n = L$, THEN $\lim_{n \to \infty} a_{n+1} = L$.

Definition of limit of a sequence

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

1. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
2. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n > n_0 \implies |L - a_n| < \varepsilon$
3. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{R} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
4. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{R},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
5. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| \leq \varepsilon$
6. $\forall \varepsilon \in (0,1),\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \varepsilon$
7. $\forall \varepsilon > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{\varepsilon}$
8. $\forall k \in \mathbb{Z}^+ > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < k$
9. $\forall k \in \mathbb{Z}^+ > 0,\ \exists n_0 \in \mathbb{N} \text{ s.t. } \forall n \in \mathbb{N},\quad n \geq n_0 \implies |L - a_n| < \frac{1}{k}$

Definition of limit of a sequence (continued)

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence. Let $L \in \mathbb{R}$.
Which statements are equivalent to "$\{a_n\}_{n=0}^{\infty} \longrightarrow L$"?

10. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all the elements of the sequence, except the first few.
11. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains all the elements of the sequence, except finitely many.
12. $\forall \varepsilon > 0$, the interval $(L - \varepsilon, L + \varepsilon)$ contains infinitely many of the terms of the sequence.
13. Every interval that contains L must contain all but finitely many of the terms of the sequence.
14. Every open interval that contains L must contain all but finitely many of the terms of the sequence.