- Today's lecture will assume you have watched videos 13.8, 13.9

  For Monday's lecture, watch videos 13.10, 13.11, 13.12, 13.13, 13.14

Rapid fire review: convergent or divergent?

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$
4. $\displaystyle\int_0^1 \frac{1}{x^2}\,dx$
5. $\displaystyle\int_0^1 \frac{1}{\sqrt{x}}\,dx$
6. $\displaystyle\int_0^1 \frac{1}{x^2+\sqrt{x}}\,dx$
7. $\displaystyle\int_0^\infty \frac{3}{x^2}\,dx$
8. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\displaystyle\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

Geometric series

Given a real number r, the series

$$\sum_{n=0}^{\infty} r^n$$

is called a geometric series. These are series we can evaluate explicitly.

Recall the following result from one of the videos:

Theorem

The geometric series $\sum_{n=0}^{\infty} r^n$ *converges if and only if* $|r| < 1$.

In this case, $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$.

Geometric series

Calculate the value of the following series:

1. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\dots$
2. $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\dots$
3. $\frac{3}{2}-\frac{9}{4}+\frac{27}{8}-\frac{81}{16}+\dots$
4. $1+\frac{1}{2^{0.5}}+\frac{1}{2}+\frac{1}{2^{1.5}}+\frac{1}{2^2}+\frac{1}{2^{2.5}}+\dots$
5. $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{2^{2n+1}}$
6. $\sum_{n=k}^{\infty} r^n$

Series expansion

From the geometric series, we know that when $|x| < 1$, we can expand the function $f(x) = \dfrac{1}{1-x}$ as a series:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Find a similar way to write the following functions as series:

$$A(x) = \frac{1}{1+x} \qquad B(x) = \frac{1}{1+x^2} \qquad C(x) = \frac{1}{2-x}$$

A challenging problem.

Problem. Calculate the value of the following series:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\,3^n}.$$

Here are some steps to guide you.

1. Compute $\sum_{n=0}^{\infty} (-1)^n x^{2n}$, for $|x| < 1$.
   (*Hint:* It's a geometric series.)
2. Compute $\frac{d}{dx}\left[\arctan x\right]$.
3. Pretend you can take derivatives and antiderivatives of series the way you can take them of sums. Which series adds up to $\arctan x$?
4. Now attempt the original problem.