- *Reminder:* Test 3 is this Thursday.
    - See the course website for info about Test 3, including what material it will cover.

- Today's lecture will assume you have watched videos 10.1-10.2

  For Tuesday's lecture, watch videos 11.1-11.2

Irreducible quadratics

1. Calculate $\int \frac{1}{x^2+1}\,dx$ and $\int \frac{x}{x^2+1}\,dx$.
   Hint: These two are very short.
2. Calculate $\int \frac{2x+3}{x^2+1}\,dx$
3. Calculate $\int \frac{x^2}{x^2+1}\,dx$
4. Calculate $\int \frac{2x+1}{x^2+x+1}\,dx$
5. Calculate $\int \frac{x}{x^2+x+1}\,dx$
   Hint: Transform it into one like the previous ones

An equation for volumes by “slicing” (like a carrot!)

You saw a specific example of this in the videos. Now let’s do it in general!

Problem. Let $a < b$ be real numbers.
Let f be a continuous, positive function defined on $[a, b]$.
Let A be the region in the first quadrant bounded between the graph of f and the x-axis.

Find a formula for the volume of the solid of revolution obtained by rotating the region A around the x-axis.

The volume of a sphere

You already know a formula for the volume of a sphere with radius R. Now you can prove it!

1. Write the equation for a circle with radius R centered at the origin.
2. If you rotate this circle around the x-axis, it will produce a sphere. Compute its volume as an integral by slicing it (*like a carrot!*).

The volume of a pyramid

Problem. Compute the volume of a pyramid with height H and square base with side length L.

Hint 1: Slice the pyramid (*like a carrot!*) with cuts parallel to the base.

Hint 2: You may need to think about similar triangles.

Let A be the region in the first quadrant bounded between the curves with equations $y = x^3$ and $y = \sqrt{32x}$.

Compute the volume of the solid of revolution obtained by rotating A around...

1. ...the x-axis
2. ...the y-axis
3. ...the line $y = -1$

(Just set up the integrals. No need to do the computation once you know you can do it.)

Volumes by "cylindrical shells" (like a volcano!)

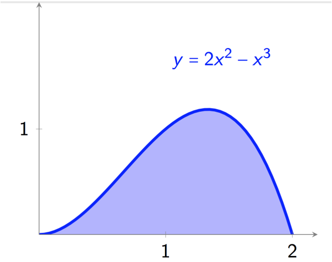
You also saw a specific example of this in the videos. Now let's do it in general!

Problem. Let $a < b$ be real numbers.
Let f be a continuous, positive function defined on $[a, b]$.
Let A be the region in the first quadrant bounded between the graph of f and the x-axis.

Find a formula for the volume of the solid of revolution obtained by rotating the region A around the <u>y-axis</u>.

A volcano!

Consider the region A between the curve $y = 2x^2 - x^3$ and the x-axis in the first quadrant:



- Rotate A around the y-axis, and compute the volume of the resulting solid. (You can use your formula from the last slide!)
- Rotate A around the line $x = -7$, and compute the volume of the resulting solid.

(Just set up the integrals.)