MAT137 - Week 12 Lecture 1

- **Reminder:** The make up lecture is taking place on Thursday December 5 at 5:00 - 7:00 in MP203.
- Please fill out the feedback form by Friday December 13. Your responses are anonymous. The link is available in the announcement titled “Mid-year course feedback”.
- Today’s lecture will assume you have watched videos 6.4, 6.5, 6.7, 6.8, 6.10

  For Tuesday’s lecture, watch videos 6.11, 6.12.

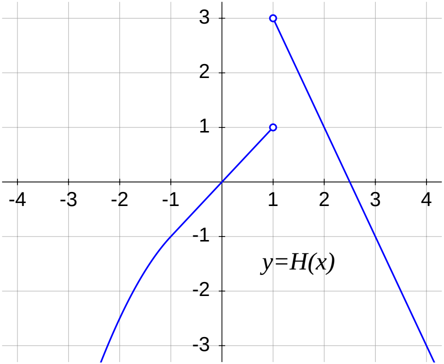
Computations

Calculate:

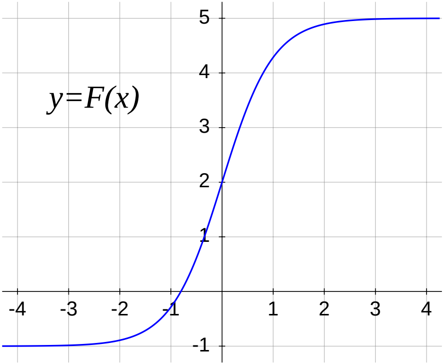
1. $\lim_{x\to 2} \frac{x^2+2x-6}{x^2+3x-10}$
2. $\lim_{x\to 0} \frac{e^{2x^2}-\cos x}{x\sin x}$
3. $\lim_{x\to \infty} \frac{x^2}{e^x}$
4. $\lim_{x\to \infty} \frac{e^x+e^{-x}}{e^x-e^{-x}}$
5. $\lim_{x\to \infty} (\sin x)\left(e^{1/x}-1\right)$
6. $\lim_{x\to \infty} x\sin\frac{2}{x}$
7. $\lim_{x\to \infty} x\cos\frac{2}{x}$
8. $\lim_{x\to 1} \left[(\ln x)\tan\frac{\pi x}{2}\right]$

Compute:

1. $\lim_{x\to 0} \frac{H(x)}{H(2+3x)-1}$



2. $\lim_{x\to 2} \frac{F^{-1}(x)}{x-2}$



Careful with L'Hôpital's Rule

Compute:

$$\lim_{x \to \infty} \frac{x + \sin x}{x}$$

What is wrong with this proof?

INCORRECT PROOF

This is indeterminate form of type $\frac{\infty}{\infty}$. So, by L'Hôpital's Rule,

$$\lim_{x \to \infty} \frac{x + \sin x}{x} = \lim_{x \to \infty} \frac{1 + \cos x}{1} = \lim_{x \to \infty} (1 + \cos x)$$

The last limit doesn't exist and so $\lim_{x \to \infty} \frac{x + \sin x}{x}$ doesn't exist.

True or False

Let f be a differentiable function on $\mathbb{R}$.

True or False: If $\lim_{x \to \infty} f(x)$ exists, then $\lim_{x \to \infty} f'(x) = 0$.

To correct the theorem, we need another assumption.
Prove this theorem:

Theorem

Let f be a differentiable function on $\mathbb{R}$. IF

- $\lim_{x \to \infty} f(x)$ exists
- $\lim_{x \to \infty} f'(x)$ exists

THEN

$$\lim_{x \to \infty} f'(x) = 0$$

Taylor polynomial preview

Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable as many times needed.

1. Find the *only* degree one polynomial P_1 such that
$$\lim_{x \to a} \frac{f(x) - P_1(x)}{x - a} = 0$$
2. Find the *only* degree two polynomial P_2 such that
$$\lim_{x \to a} \frac{f(x) - P_2(x)}{(x - a)^2} = 0$$
3. Let $n \in \mathbb{N}$. Find the *only* degree n polynomial P_n such that
$$\lim_{x \to a} \frac{f(x) - P_n(x)}{(x - a)^n} = 0$$

What does this mean about f and P_n?
Which P_n approximates f best near a? (which of (1) or (2) is stronger?)