MAT137 - Week 4 Lecture 2

- Today's lecture will assume you have watched videos 2.12 - 2.13
  For Monday's lecture, watch videos 2.14 - 2.18.

Warm up for a difficult $\varepsilon - \delta$ proof

Let $\varepsilon > 0$.

Find one positive value δ such that

$$|x| < \delta \implies |x(x+1)| < \varepsilon$$

A more complicated $\varepsilon - \delta$ proof.

Problem. Prove, directly from the formal definition of the limit, that

$$\lim_{x \to 1} \frac{1}{x^2 + 1} = \frac{1}{2}.$$

First:

1. Write down the formal definition of what you're trying to prove.
2. Write down what the structure of the proof should be, without filling in any details. What variables must you define in what order, what must you assume and where, etc.
3. Figure out what δ should be given ε (that will be your rough work). Then fill in the details and complete the proof.

Is this theorem True or False

We have shown that $\forall \varepsilon > 0$, $\exists \delta > 0$ such that

$$|x| < \delta \implies |x(x+1)| < \varepsilon$$

i.e. we have shown that $\lim_{x \to 0} x(x+1) = 0$.

Is this generally true? Can we use the limit laws?

Theorem

Let g be any function defined on $\mathbb{R}$. Then the following is always true:

$$\lim_{x \to 0} xg(x) = 0$$

The Correct Generalization

Theorem

Let g be any function defined on $\mathbb{R}$, except possibly at 0. If g is bounded on an interval centered at 0, except possibly at 0, then

$$\lim_{x \to 0} xg(x) = 0$$

Prove it!