

Graduate course MAT 1347 HF

SYMPLECTIC TOPOLOGY and MORSE THEORY

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Fall 2014: W,F 11am-1pm

SYLLABUS

(the numbers below approximately correspond to the week numbers):

- 1) Preliminaries/reminder: Symplectic manifolds, Hamiltonian fields, Darboux theorem, Lagrangian manifolds and foliations, integrable systems.
- 2) Symplectic properties of billiards and, time permitted, geodesics on an ellipsoid.
- 3-4) Symplectic fixed points theorems: the Poincaré–Birkhoff theorem, Arnold’s conjecture, the Conley–Zehnder theorem.
- 4-6) Morse theory: Morse inequalities, Lusternik–Schnirelmann category, applications to geodesics, other ramifications (the Morse–Witten complex, Morse–Novikov theory); the end of proof for Conley–Zehnder.
- 6-7) A glimpse of generating functions for symplectomorphisms, non-squeezing results, symplectic capacities, Floer homology.
- 7-8) The Hofer metric, geometry of and geodesics on symplectomorphism groups.
- 8-9) Contact structures, Legendrian knots, their invariants and Bennequin inequality; a glimpse of contact homology of Legendrian knots.

References:

1. S. Tabachnikov, "Introduction to symplectic topology" Lecture notes, (PennState U.): <http://www.math.psu.edu/tabachni/courses/symplectic.pdf>
2. D. McDuff and D. Salamon: "Introduction to symplectic topology" (Oxford Math. Monographs, 1998)
3. V. Arnold and A. Givental "Symplectic geometry" Dynamical systems, IV, 1–138, Encyclopaedia Math. Sci., vol. 4, (Springer 2001)

Prerequisite:

A basic course in symplectic geometry (or familiarity with its main notions).