Graduate course MAT 1347HS

SYMPLECTIC TOPOLOGY AND INTEGRABLE SYSTEMS

B. Khesin

Spring 2008

SYLLABUS

(the numbers below approximately correspond to the week numbers):

1) Preliminaries/reminder: Symplectic manifolds, Hamiltonian fields, Darboux theorem, Lagrangian manifolds and foliations, integrable systems.

2) Symplectic properties of billiards and, time permitted, geodesics on an ellipsoid.

3-4) Symplectic fixed points theorems: the Poincare–Birkhoff theorem, Arnold's conjecture, the Conley–Zehnder theorem.

5-6) Morse theory: Morse inequalities, Lusternik-Schnirelmann category, applications to geodesics, other ramifications (the Morse–Witten complex, Morse–Novikov theory); the end of proof for Conley–Zehnder.

7) A glimpse of generating functions for symplectomorphisms, non-squeezing results, symplectic capacities, Floer homology.

8) The Hofer metric, geometry of and geodesics on symplectomorphism groups.

9-10) Contact structures, Legendrian knots, their invariants and Bennequin inequality; a glimpse of contact homology of Legendrian knots.

11) The Lie–Poisson bracket, compatible brackets, the shift argument method, integrability.

12) Toda lattices and the KdV equation.

References:

1. S. Tabachnikov, "Introduction to symplectic topology" Lecture notes, (PennState U.): http://www.math.psu.edu/tabachni/courses/symplectic.pdf

2. D. McDuff and D. Salamon: "Introduction to symplectic topology" (Oxford Math. Monographs, 1998)

3. V. Arnold and A. Givental "Symplectic geometry" Dynamical systems, IV, 1–138, Encyclopaedia Math. Sci., vol. 4, (Springer 2001)

Prerequisite:

A basic course in symplectic geometry (or familiarity with its main notions).