Instructor: B. Khesin

Course MAT461S
Spring 2025
"Hamiltonian Mechanics"

Problem Set 3 (due Thursday Mar. 20):

All 4 problems count (5pts each).

Main source: [Ar] textbook by V. Arnold, see

`https://www.math.toronto.edu/khesin/biblio/arnold_Math_Methods89.pdf`

1. The Lagrangian for a system of one degree of freedom can be written as

$$L = \frac{m}{2}(\dot{q}^2 \sin^2 \omega t + \dot{q}q\omega \sin 2\omega t + q^2\omega^2)\,.$$

a) What is the corresponding Hamiltonian? Is it conserved? b) Introduce a new coordinate defined by $Q = q \sin \omega t$. Find the Lagrangian in terms of the new coordinate and the corresponding Hamiltonian. Is the new Hamiltonian conserved?

2. What is the height-to-diameter ratio of a right cylinder such that the inertia ellipsoid at the center of the cylinder is a sphere?

3. Find the principal moments of inertia about the center of mass of a flat rigid body in the shape of the 45-45-90–degree right triangle with uniform mass density. What are its principal axes of inertia?

4. A particle is thrown up vertically with initial speed v_0, reaches a maximum height and falls back to ground. Show that the Coriolis deflection when it again reaches the ground is opposite in direction, and four times greater in magnitude, than the Coriolis deflection when it is dropped at rest from the same maximum height.

Bonus problem:

(1pt) Sketch the inertial ellipsoid of a 1D rod fixed at its middle point.