MAT137

- Today we will discuss the definition of limit.
- Homework before Thursday's class: watch videos 2.7, 2.8, as well as 2.9.

Definition of Limit

Definition

Let a, L be two real numbers. Let f be a function defined on an interval containing $a \in \mathbb{R}$, except possibly at a. We write

$$\lim_{x \to a} f(x) = L$$

when

$$\forall \varepsilon > 0 \ \exists \delta > 0 \ \text{ s.t. } \ 0 < |x - a| < \delta \Rightarrow |f(x) - L| < \varepsilon$$

Self-check

Write down the formal definition of

$$\lim_{x \to 3} (x+1)^2 = 16.$$

Side limits

We know:

Definition

Let $L, a \in \mathbb{R}$.
Let f be a function defined at least on an interval around a, except possibly at a.

$$\lim_{x \to a} f(x) = L$$

means

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ s.t. } 0 < |x - a| < \delta \implies |f(x) - L| < \varepsilon.$$

Write, instead, the formal definition of

$$\lim_{x \to a^+} f(x) = L, \quad \text{and} \quad \lim_{x \to a^-} f(x) = L.$$

Limits at infinity: Self-check

Write down the formal definition of

$$\lim_{x \to \infty} f(x) = 3.$$

Preparation: choosing deltas

1. Find a value of $\delta > 0$ such that

$$|x-3| < \delta \implies |5x-15| < 1.$$

2. Find *all* values of $\delta > 0$ such that

$$|x-3| < \delta \implies |5x-15| < 1.$$

3. Find a value of $\delta > 0$ such that

$$|x-3| < \delta \implies |5x-15| < 0.1.$$

4. Let us fix $\varepsilon > 0$. Find a value of $\delta > 0$ such that

$$|x-3| < \delta \implies |5x-15| < \varepsilon.$$

Your first $\varepsilon - \delta$ proof

Goal

We want to prove that

$$\lim_{x \to 3} (5x + 1) = 16 \tag{1}$$

directly from the definition.

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1. Write down the formal definition of the statement (1).

Your first $\varepsilon - \delta$ proof

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1. Write down the formal definition of the statement (1).
2. Write down what the structure of the formal proof should be, without filling the details.

Your first $\varepsilon - \delta$ proof

Goal

We want to prove that

$$\lim_{x \to 3} (5x + 1) = 16 \tag{1}$$

directly from the definition.

1. Write down the formal definition of the statement (1).
2. Write down what the structure of the formal proof should be, without filling the details.
3. Write down a complete formal proof.

Proof outline

Let $\varepsilon > 0$.

Take $\delta =???$.

Let $x \in \mathbb{R}$. Assume that $0 < |x - 3| < \delta$.

... some algebra ...

Thus $|f(x) - 16| < \varepsilon$.

A harder limit

Let $f(x) = \frac{3}{x}$.

Prove that

$$\lim_{x \to 1} f(x) = 3$$