- Enrol in a tutorial if you haven't already done so. Tutorials are starting this week. You will be able to switch tutorials even after registration closes.
- Today we will discuss definitions and proofs.
- Homework before Wednesday's class: watch videos 1.14, 1.15.

Warm-up: Symmetric difference

Given two sets A and B, we define

- $A \setminus B = \{x \in A \ : \ x \notin B\}$
- $A \triangle B = (A \setminus B) \cup (B \setminus A)$

Let

- $C_1 = \{$ students in class wearing green today$\}$
- $C_2 = \{$ students sitting in the first two rows $\}$

What is the set $C_1 \triangle C_2$?

Even numbers

Which of the two definitions is correct?

Definition

$n \in \mathbb{Z}$ is even if $\exists a \in \mathbb{Z}$, s.t. $n = 2a$.

OR

Definition

$n \in \mathbb{Z}$ is even if $\forall a \in \mathbb{Z}$, $n = 2a$.

What is wrong with this proof?

Theorem

The sum of two odd numbers is even.

Proof.

$x = 2a + 1$ odd
$y = 2b + 1$ odd
$x + y = 2n$ even
$2a + 1 + 2b + 1 = 2n$
$2a + 2b + 2 = 2n$
$a + b + 1 = n$ □

Write a correct proof.

One-to-one

A function is called *one-to-one* if different inputs always produce different outputs.

Write a formal definition of “one-to-one”.

After you have written your definition, exchange with your neighbour and discuss.

Examples

Which of the following functions are one-to-one?

1. x^2 on the interval $[-1, 1]$.

2. x^3 on the interval $[-1, 1]$.

3. Define $f : \mathbb{Z} \to \{0, 1\}$ by

$$f(n) = \begin{cases} 0, & \text{if } n \text{ is even} \\ 1, & \text{if } n \text{ is odd} \end{cases}$$

Possible definitions

Let f be a function with domain D. What do the following mean?

1. $f(x_1) \neq f(x_2)$
2. $\exists x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
3. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2)$
4. $\forall x_1, x_2 \in D,\ x_1 \neq x_2,\ f(x_1) \neq f(x_2)$
5. $\forall x_1, x_2 \in D,\ f(x_1) \neq f(x_2) \implies x_1 \neq x_2$
6. $\forall x_1, x_2 \in D,\ x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$
7. $\forall x_1, x_2 \in D,\ f(x_1) = f(x_2) \implies x_1 = x_2$

Increasing and one-to-one

Recall that a function f is called *increasing* on an interval I when

$$\forall x, y \in I,\ x < y \ \Rightarrow\ f(x) < f(y).$$

Let's prove the following theorem

Theorem

If f is increasing on an interval I, then it is one-to-one on the interval I.