Enrol in a tutorial if you haven’t already done so. Tutorials start today. You will be able to switch tutorials even after registration closes.

Today we will discuss definitions and proofs.

Homework before Wednesday’s class: watch videos 1.14, 1.15.
Symmetric difference

Given two sets $A$ and $B$, we define

- $A \setminus B = \{ x \in A : x \notin B \}$
- $A \triangle B = (A \setminus B) \cup (B \setminus A)$

Let

- $C_1 = \{ \text{students in class wearing green today} \}$
- $C_2 = \{ \text{students sitting in the first two rows} \}$

What is the set $C_1 \triangle C_2$?
Even numbers

Which of the two definitions is correct?

**Definition**

$n \in \mathbb{Z}$ is even if $\exists a \in \mathbb{Z}$, s.t. $n = 2a$.

**OR**

**Definition**

$n \in \mathbb{Z}$ is even if $\forall a \in \mathbb{Z}$, $n = 2a$. 
What is wrong with this proof?

Theorem
The sum of two odd numbers is even.

Proof.
\[ x = 2a + 1 \text{ odd} \]
\[ y = 2b + 1 \text{ odd} \]
\[ x + y = 2n \text{ even} \]
\[ 2a + 1 + 2b + 1 = 2n \]
\[ 2a + 2b + 2 = 2n \]
\[ a + b + 1 = n \]

Write a correct proof.
A function is called *one-to-one* if different inputs always produce different outputs.

Write a formal definition of “one-to-one”.

After you have written your definition, exchange with your neighbour and discuss.
Possible definitions

Let \( f \) be a function with domain \( D \). What do the following mean?

1. \( f(x_1) \neq f(x_2) \)
2. \( \exists x_1, x_2 \in D, f(x_1) \neq f(x_2) \)
3. \( \forall x_1, x_2 \in D, f(x_1) \neq f(x_2) \)
4. \( \forall x_1, x_2 \in D, x_1 \neq x_2, f(x_1) \neq f(x_2) \)
5. \( \forall x_1, x_2 \in D, f(x_1) \neq f(x_2) \implies x_1 \neq x_2 \)
6. \( \forall x_1, x_2 \in D, x_1 \neq x_2 \implies f(x_1) \neq f(x_2) \)
7. \( \forall x_1, x_2 \in D, f(x_1) = f(x_2) \implies x_1 = x_2 \)
Which of the following functions are one-to-one?

1. $x^2$ on the interval $[-1, 1]$.

2. $x^3$ on the interval $[-1, 1]$.

3. Define $f : \mathbb{Z} \to \{0, 1\}$ by

   $$f(n) = \begin{cases} 
   0, & \text{if } n \text{ is even} \\
   1, & \text{if } n \text{ is odd} 
   \end{cases}$$
Increasing and one-to-one

Recall that a function $f$ is called *increasing* on an interval $I$ when

$$\forall x, y \in I, \ x < y \Rightarrow f(x) < f(y).$$

Let’s prove the following theorem:

**Theorem**

*If $f$ is increasing on an interval $I$, then it is one-to-one on the interval $I$.*