Course website: Quercus and http://www.math.toronto.edu/khesin/teaching/mat137.html

Today we will discuss negation and quantifiers.

Before next class, watch videos 1.7, 1.8, as well as 1.9.
Negation

To negate a statement means to find a statement which is true exactly when the original statement is false.

What is the negation of “Everyone is this class likes chocolate ice cream”?

1. No one in this class likes chocolate ice cream.
2. There is someone in this class who doesn’t like chocolate ice cream.
3. There is someone in this class who only likes vanilla ice cream.
4. There is someone in this class who doesn’t like ice cream.
5. Everyone in this class only likes vanilla ice cream.
More negation

What are the negations of the following statements? Are the following sentences true?

1. Every person in this class has a friend who was born outside of Canada.
2. For every $x \in \mathbb{Z}$, there exists $y \in \mathbb{R}$ such that $y^2 = x$.
3. For every $n \in \mathbb{Z}$, there exists $m \in \mathbb{Z}$ such that $m > n$.
4. There exists $x \in \mathbb{R}$ such that $x^3 < 0$.
5. There exists $x \in \mathbb{R}$ such that for all $y \in \mathbb{R}$, $x > y$. 
1. There exists a pink elephant in this room.

2. All elephants in this room are pink.

3. For all $x \in [0, 2] \cap [4, 8]$, we have that $x^2 < 0$.

4. There exists $x \in [0, 4] \cap [2, 8]$, such that $x^2 < 0$.

5. There exists $x \in [0, 2] \cap [4, 8]$, such that $x^2 < 0$. 
For all $a \in (0, \infty)$, there exists $b \in (0, \infty)$, such that for all $c \in (-b, b)$, we have $c^2 < a$. Indeed, for an arbitrary $a \in (0, \infty)$ take $b = \sqrt{a}$. Then for any $c \in (-b, b)$, i.e. for any $c$ satisfying $-\sqrt{a} < c < \sqrt{a}$, one has $c^2 < a$. 
A complicated statement: True or False

For all $a \in (0, \infty)$, there exists $b \in (0, \infty)$, such that for all $c \in (-b, b)$, we have $c^2 < a$.

True. Indeed, for an arbitrary $a \in (0, \infty)$ take $b = \sqrt{a}$. Then for any $c \in (-b, b)$, i.e. for any $c$ satisfying $-\sqrt{a} < c < \sqrt{a}$, one has $c^2 < a$. 