MAT137

- Course website: Quercus and
  `http://www.math.toronto.edu/khesin/teaching/mat137.html`
- Problem set 1 is on GradeScope now. Please, submit all of your assignments on GradeScope.
- For Instructions (but not submission!) see Quercus.
- Enrol in a tutorial!
- Today we will discuss sets and basic quantifiers.
- **Before next class, watch videos 1.4, 1.5, 1.6.**

Sets

What are the following sets?

1. $[1,3] \cup [2,6]$
2. $[1,3] \cap (2,6)$
3. $[1,3] \cap (3,7)$
4. $[2,2]$
5. $[7,3]$
6. $(2,2)$

More sets

What are the following sets?

1. $\{x \in \mathbb{R} \ : \ x^2 < 4\}$
2. $\{x \in \mathbb{R} \ : \ \exists y \in [0,1] \text{ such that } y < x\}$
3. $\{x \in \mathbb{R} \ : \ \forall y \in [0,1], \ y < x\}$
4. $\{x \in \mathbb{R} \ : \ x^2 + 5 < 3\}$
5. $\{x \in \mathbb{R} \ : \ \exists y \in \mathbb{R} \text{ s. t. } y^2 = x\}$
6. $\{x \in \mathbb{R} \ : \ \forall y \in \mathbb{R}, \ y^2 = x\}$
7. $\{x \in \mathbb{R} \ : \ \exists y, z \in \mathbb{Z} \text{ s. t. } xy = z\}$
8. $\{x \in \mathbb{R} \ : \ \exists y, z \in \mathbb{Z} \text{ s. t. } xy = z \text{ and } y \neq 0\}$

Yet more sets

Express the following sets using set building notation.

1. The set of all real numbers whose cube is an integer.
2. The set of all real numbers which can be written as the square of a rational number.

True or False

1. $[1,3] \subseteq (1,4)$
2. $3 \in (1,5]$
3. $\mathbb{N} \subseteq \mathbb{R}$
4. $\mathbb{Q} \subset \mathbb{Z}$
5. $3 \in \{x \in \mathbb{R} \ : \ \exists y \in \mathbb{N} \text{ s. t. } y^2 = x\}$
6. $3 \in \{x \in \mathbb{R} \ : \ \exists y \in \mathbb{N} \text{ s. t. } y^2 < 2 \text{ and } y < x\}$
7. $3 \in \{x \in \mathbb{R} \ : \ \exists y \in \mathbb{N} \text{ s. t. } y^2 = 2 \text{ and } y < x\}$

Algebraic numbers

Q.: A real number is called algebraic if it is the root of a polynomial equation with integer coefficients. Express the set A of algebraic numbers using set builder notation.

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Q.: A real number is called algebraic if it is the root of a polynomial equation with integer coefficients. Express the set A of algebraic numbers using set builder notation.

A.:

$$A = \{x \in \mathbb{R} \ : \ \exists n \in \mathbb{N} \text{ and } \exists a_0, a_1, ..., a_n \in \mathbb{Z},$$
$$\text{s.t. } a_0 + a_1 x + a_2 x^2 + ... + a_n x^n = 0\}$$