Today we start discussing continuity.

Homework before Wednesday’s class: watch videos 2.16, 2.17, as well as 2.18.
Let $f(x)$ be a function. Suppose that $f(0) = 2$, but $f(x) < 0$ for all other $x \in (-1, 1)$. What can we conclude?

1. $\lim_{x \to 0} f(x)$ does not exist.
2. $f(x)$ is not continuous at 0.
3. We cannot conclude anything.
Find an example of a function $f(x)$ such that $f(x)$ is defined on $\mathbb{R}$ and satisfies:

1. $f(x)$ is continuous at every $c \in \mathbb{R}$.
2. $f(x)$ is continuous at every $c \in \mathbb{R} \setminus \{0\}$ and is not continuous at 0.
3. $f(x)$ is discontinuous at every $c \in \mathbb{R}$.
4. $f(x)$ is continuous at 0 and discontinuous at every other $c \in \mathbb{R}$.

Hint: remember the Dirichlet function

$$f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$
The following functions are not defined at 0. How should we define them at 0, so that they will be continuous?

1. \(x^2 \sin(1/x) + 5\)

2. \(\frac{x}{\sqrt{x + 1} - 1}\)

3. \(\frac{x - 1/x}{1 + 3/x}\)
A difficult example

Is it possible to construct functions such that....?

1. \( \lim_{x \to 1} f(x) = 2 \)
2. \( \lim_{u \to 2} g(u) = 3 \)
3. \( \lim_{x \to 1} g(f(x)) = 42 \)