- Today we will discuss limit laws.
- Homework before Wednesday's class: watch videos 2.12, 2.13.

Proof of non-existence

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We want to prove that

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2. Write down what the structure of the formal proof should be, without filling the details.
3. Write down a complete formal proof.

Uniqueness of limits

Theorem

Let a, L_1, L_2 *be real numbers. Let* f *be a function defined on an interval containing* $a \in \mathbb{R}$*, except possibly at* a*. Suppose that*

$$\lim_{x \to a} f(x) = L_1 \quad \textit{and} \quad \lim_{x \to a} f(x) = L_2$$

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Then $L_1 = L_2$.

By contradiction: assume that $L_1 \neq L_2$. Then take $\varepsilon = |L_1 - L_2|/3 > 0$. Then $\exists \delta > 0$ s.t. ...

Some strange guys

Can you find functions $f(x)$ and $g(x)$ with the following properties?

1. $\lim_{x\to 0} f(x)$ does not exists and $\lim_{x\to 0}(f(x)+g(x)) = 0$.
2. $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} f(x)g(x) = 14$.
3. $\lim_{x\to 0} f(x)$ does not exist and $\lim_{x\to 0} g(x)$ does not exist, but $\lim_{x\to 0} f(x)g(x) = 0$.
4. $\lim_{x\to 0} f(x) = 2$ and $\lim_{x\to 0} g(x) = 3$, but $\lim_{x\to 0} g(f(x)) = 14$.

Indeterminate form

Let $a \in \mathbb{R}$.
Let f and g be functions defined near a.
Assume $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$.

What can we conclude about $\lim_{x \to a} \frac{f(x)}{g(x)}$?

1. The limit is 1.
2. The limit is 0.
3. The limit is ∞.
4. The limit does not exist.
5. We do not have enough information to decide.