Today: Derivatives of exponentials and logarithms

Homework before Wednesday’s class: watch videos 3.19, 3.20.
Compute the derivative of the following functions:

1. \( f(x) = e^{\sin x + \cos x} \ln x \)
2. \( f(x) = \pi^{\tan x} \)
3. \( f(x) = \ln [e^x + \ln \ln \ln x] \)

Reminder: We know:

\[
\begin{align*}
\frac{d}{dx} e^x &= e^x \\
\frac{d}{dx} a^x &= a^x \ln a \\
\frac{d}{dx} \ln x &= \frac{1}{x}
\end{align*}
\]
A different type of logarithm

Calculate the derivative of

\[ f(x) = \log_{x+1}(x^2 + 1) \]

*Hint:* If you do not know where to start, remember the definition of logarithm:

\[ \log_a b = c \iff a^c = b. \]
Calculate the derivative of

\[ g(x) = x^{\tan x}. \]
Calculate the derivative of

\[ f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x} . \]
More logarithmic differentiation

Calculate the derivative of

\[ f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}. \]

What is wrong with this answer?

\[
\ln f(x) = (\cos x) \ln(\sin x) + (\sin x)(\ln \cos x)
\]

\[
\frac{d}{dx} \left[ \ln f(x) \right] = \frac{d}{dx} \left[ (\cos x) \ln(\sin x) \right] + \frac{d}{dx} \left[ (\sin x)(\ln \cos x) \right]
\]

\[
\frac{f'(x)}{f(x)} = - (\sin x) \ln(\sin x) + (\cos x) \frac{\cos x}{\sin x}
\]

\[ + (\cos x) \ln(\cos x) + (\sin x) \frac{\sin x}{\cos x} \]

\[
f'(x) = f(x) \left[ - (\sin x) \ln(\sin x) + (\cos x) \ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x} \right]
\]
Calculate the derivative of

\[
h(x) = \sqrt[3]{\frac{\sin^6 x \sqrt{x^7 + 6x + 2}}{3^x (x^{10} + 2x)^{10}}}
\]