MAT137

- Today: Proof of the differentiation rules.
- Homework before Tuesday's class: watch videos 3.10, 3.11.

True or False

Let $a \in \mathbb{R}$.
Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable everywhere.
What can we conclude?

1. $f(a)$ is defined.
2. $\lim_{x \to a} f(x)$ exists.
3. f is continuous at a.
4. $f'(a)$ exists.
5. $\lim_{x \to a} f'(x)$ exists.
6. f' is continuous at a.

A continuity lemma

Write a formal proof for:

Lemma

Let $a \in R$.
Let g be a function continuous at a.
Assume that $g(a) \neq 0$. Then $g(x) \neq 0$ for x close to a.

Note: First, figure out what "$g(x) \neq 0$ for x close to a" means formally.

Write a formal proof for the quotient rule for derivatives

Theorem

- Let $a \in \mathbb{R}$.
- Let f and g be functions defined at and near a. Assume $g(a) \neq 0$.
- We define the function h by $h(x) = \dfrac{f(x)}{g(x)}$.

IF f and g are differentiable at a,

THEN h is differentiable at a, and

$$h'(a) = \frac{f'(a)g(a) - f(a)g'(a)}{g(a)^2}.$$

Write a proof directly from the definition of derivative.
Hint: Imitate the proof of the product rule in Video 3.6.

Check your proof

- Are there words or only equations?
- Does every step follow logically?
- Did you only assume things you could assume?

- At some point in your proof you must have used, for example, that g was continuous. (Otherwise your proof is most likely wrong.)
  Did you notice you were using this? Did you justify it?

Critique this proof

$$h'(a) = \lim_{x\to a} \frac{h(x)-h(a)}{x-a} = \lim_{x\to a} \frac{\frac{f(x)}{g(x)} - \frac{f(a)}{g(a)}}{x-a}$$

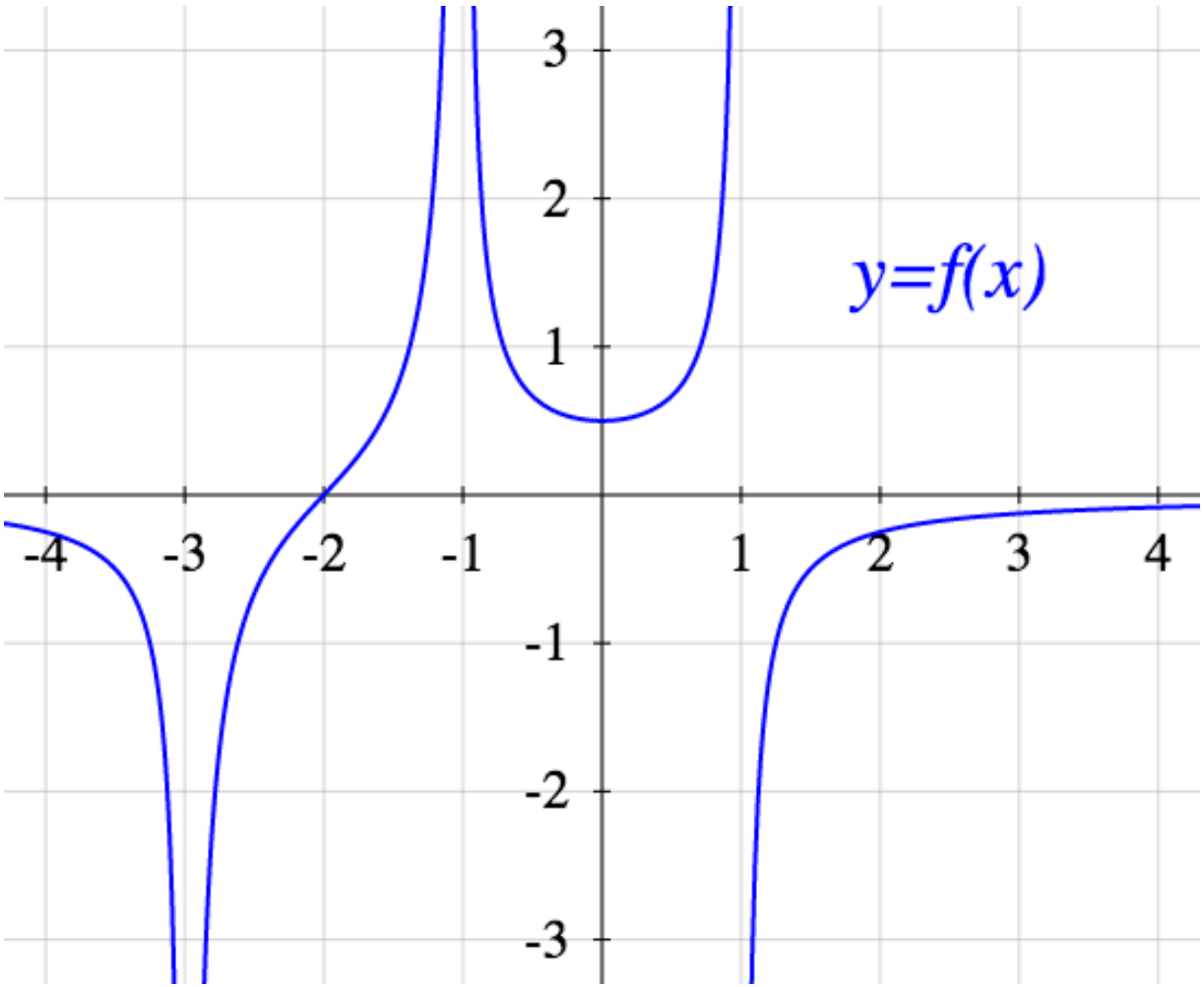
$$= \lim_{x\to a} \frac{f(x)g(a)-f(a)g(x)}{g(x)g(a)\,(x-a)}$$

$$= \lim_{x\to a} \frac{f(x)g(a)-f(a)g(a)+f(a)g(a)-f(a)g(x)}{g(x)g(a)\,(x-a)}$$

$$= \lim_{x\to a} \left\{ \left[\frac{f(x)-f(a)}{x-a} g(a) - f(a) \frac{g(x)-g(a)}{x-a} \right] \frac{1}{g(x)g(a)} \right\}$$

$$= \left[f'(a)g(a) - f(a)g'(a) \right] \frac{1}{g(a)g(a)}$$

Reminder: What is the equation for this function? $f(x) = \ldots$



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Vertical things

Construct a function that has both

- a vertical tangent line at $x = 1$, and
- a vertical asymptote at $x = -1$.

From the derivative to the function

1. Sketch the graph of a continuous function whose derivative has the graph below
2. Sketch the graph of a non-continuous function whose derivative has the graph below