MAT137

- Today: Definition of derivative.

- Homework before Wednesday class: watch videos 3.4, 3.5, 3.8.

From last time: More uses of IVT

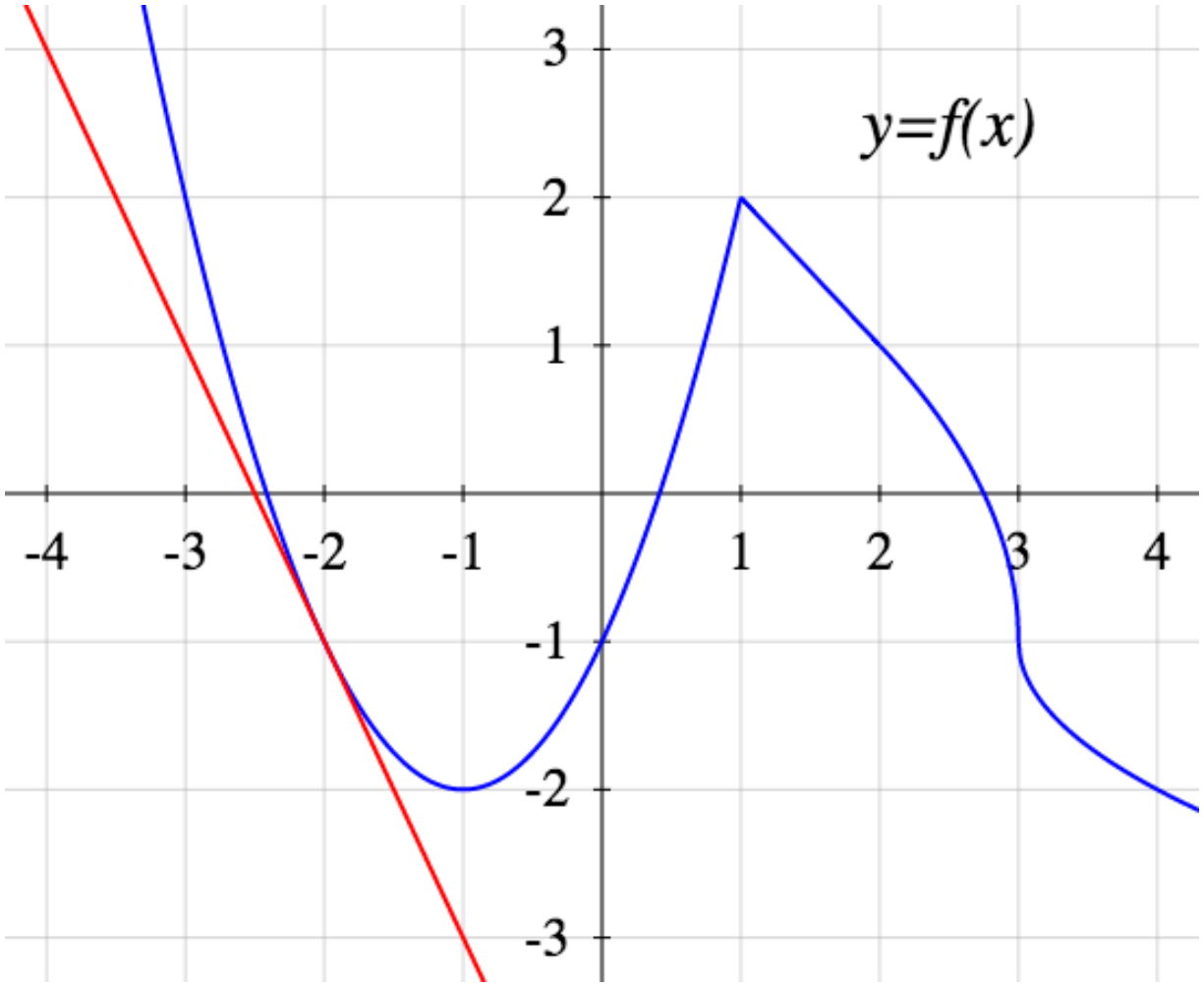
1. Prove that the equation $x^3 + \sin x - 1 = 0$ has a solution.

2. Prove that the equation $x^4 - 2x = 100$ has at least two solutions.

3. Suppose that $f(x)$ is a continuous function on $[0, 1]$ such that $f(x) \in [0, 1]$. Prove that the equation $f(x) = x$ has a solution.

Tangent line from a graph

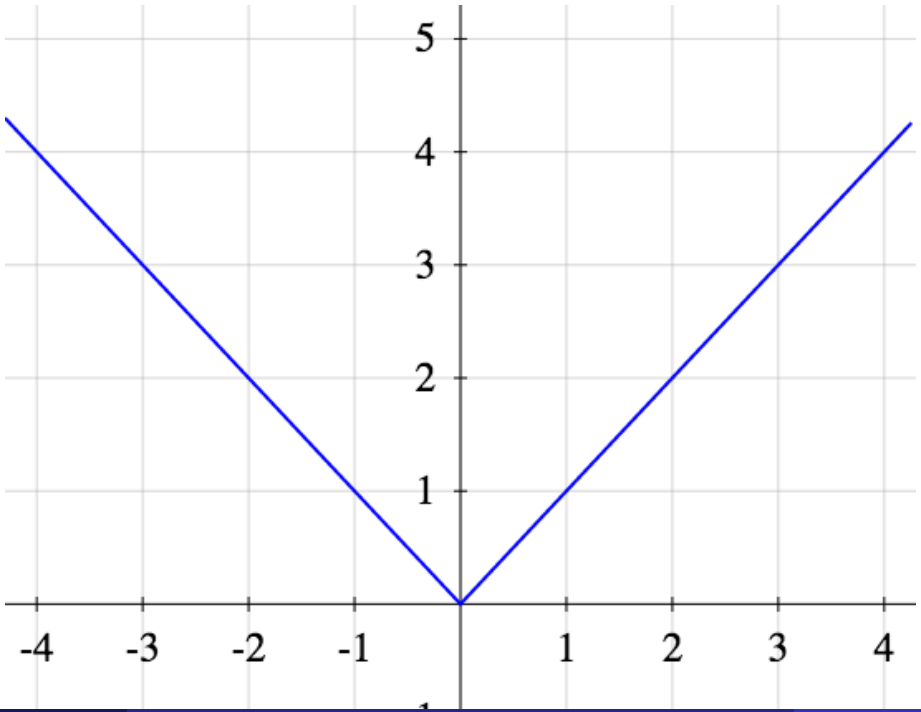
Below is the graph of the function f. Write the equation of the line tangent to it at the point with x–coordinate -2.



Absolute value and tangent lines

At (0,0) the graph of $y = |x|$...

1. ... has one tangent line: $y = 0$
2. ... has one tangent line: $x = 0$
3. ... has two tangent lines $y = x$ and $y = -x$
4. ... has no tangent line



The definition

Let $f(x)$ be a function defined on some interval containing a. We say that $f(x)$ is *differentiable at* a when

$$\lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

exists. Notation for the limit: $f'(a)$.

Meaning of derivative $f'(a)$:

- the slope of the tangent to the graph $y = f(x)$ at $x = a$.
- the instantaneous rate of change of $f(x)$ at $x = a$.

Absolute value and derivatives

Let $g(x) = x|x|$. What is $g'(0)$?

1. It is 0.
2. It does not exist because $|x|$ is not differentiable at 0.
3. It does not exist because the right- and left-limits, when computing the derivative, are different.
4. It does not exist because it has a corner.

Derivatives from the definition

Let

$$g(x) = \frac{2}{\sqrt{x}}$$

Calculate $g'(4)$ directly from the definition of derivative as a limit.

Derivative from a graph

Below is the graph of the function f.
Sketch the graph of its derivative f'.