Test 1 is this Friday, 4-6pm.

Today: IVT and EVT.

Homework before Wednesday’s class: watch videos 3.1, 3.2, 3.3.
Do the following functions have a maximum on the given interval?

1. \( x \) on \([1, \infty)\).
2. \( \frac{1}{x} \) on \((0, 1)\).
3. \( \frac{1}{x} \) on \([1, \infty)\).
4. \( e^{5x^2+37x} \sin(4x)^2 + e^{27x^3} \cos x \) on \([1, 2]\).
Extreme Value Theorem

Let \( f(x) \) be a continuous function on a closed interval \([a, b]\). Then \( f(x) \) has a maximum and a minimum on \([a, b]\).

1. Find an example of \( f(x) \) which is not continuous and does not have a maximum on \([0, 1]\).
2. Find an example of a continuous \( f(x) \) which does not have a maximum on \((0, 1)\).
In each of the following cases, does the function $f$ have a maximum and a minimum on the interval $I$?

1. $f(x) = x^2$, $I = (-1, 1)$.
2. $f(x) = \frac{(e^x + 2) \sin x}{x} - \cos x + 3$, $I = [2, 6]$
3. $f(x) = \frac{(e^x + 2) \sin x}{x} - \cos x + 3$, $I = [-2, 2]$
Cinderella weighed 7 pounds when she was born and she weighed 110 pounds when she died at age 120. Was there ever a point in her life at which her weight in pounds equalled her age in years?
Intermediate Value Theorem

Theorem

Let $f(x)$ be a continuous function on a closed interval $[a, b]$. Let $K \in \mathbb{R}$. If $f(a) \leq K \leq f(b)$, then there exists $c \in [a, b]$ such that $f(c) = K$. 

How to solve the Cinderella question by using the IVT?

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MAT137
October 15, 2018
Intermediate Value Theorem

**Theorem**

Let \( f(x) \) be a continuous function on a closed interval \([a, b]\). Let \( K \in \mathbb{R} \).

If \( f(a) \leq K \leq f(b) \), then there exists \( c \in [a, b] \) such that \( f(c) = K \).

How to solve the Cinderella question by using the IVT?
More uses of IVT

1. Prove that the equation $x^3 + \sin x - 1 = 0$ has a solution.
More uses of IVT

1. Prove that the equation $x^3 + \sin x - 1 = 0$ has a solution.

2. Prove that the equation $x^4 - 2x = 100$ has at least two solutions.
1. Prove that the equation $x^3 + \sin x - 1 = 0$ has a solution.

2. Prove that the equation $x^4 - 2x = 100$ has at least two solutions.

3. Suppose that $f(x)$ is a continuous function on $[0, 1]$ such that $f(x) \in [0, 1]$. Prove that the equation $f(x) = x$ has a solution.
Definition of maximum

Let $f$ be a function with domain $I$.
What does each of the following mean?

1. $\forall x \in I, \exists C \in \mathbb{R}$ s.t. $f(x) \leq C$
2. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) \leq C$
3. $\exists C \in \mathbb{R}$ s.t. $\forall x \in I, f(x) < C$
4. $\exists a \in I$ s.t. $\forall x \in I, f(x) \leq f(a)$
5. $\exists a \in I$ s.t. $\forall x \in I, f(x) < f(a)$