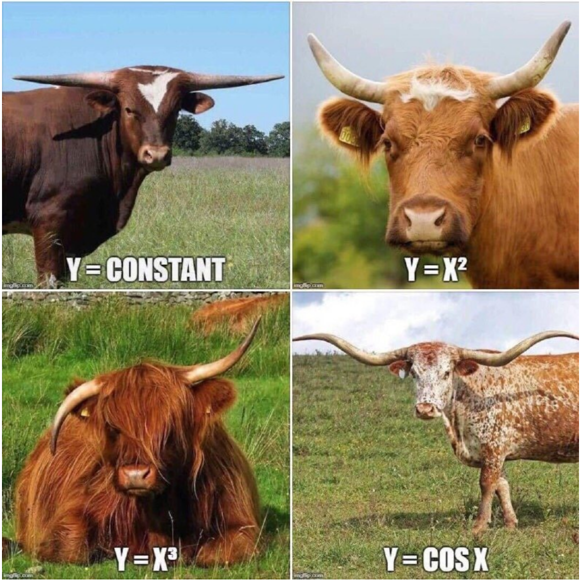
- Today: the remarkable limit and continuity of composition.

- Homework before Wednesday's class: watch videos 2.19, 2.20.

Elementary continuous functions



A difficult example

Is it possible to construct functions such that....?

1. $\lim_{x \to 1} f(x) = 2$
2. $\lim_{u \to 2} g(u) = 3$
3. $\lim_{x \to 1} g(f(x)) = 42$

True or False?

(Assuming these limits exist)

$$\lim_{x \to a} g(f(x)) = g\left(\lim_{x \to a} f(x)\right)$$

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(Assuming these limits exist)

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What extra condition do we need to add for this to be true?

A composition theorem

Theorem

Let $a, L \in \mathbb{R}$.
Let f and g be functions.
IF

- $\lim_{x \to a} f(x) = L$
- g is continuous at L

THEN $\lim_{x \to a} g(f(x)) = g(L)$

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How can we prove this?

$|g(f(x)) - g(L)| < \varepsilon$

A new function

Let $x, y \in \mathbb{R}$. What does the following expression calculate? Prove it.

$$f(x, y) = \frac{x + y + |x - y|}{2}$$

Suggestion: If you don't know how to start, try some sample values of x and y.

Write a similar expression to compute $\min\{x, y\}$.

More continuous functions

We want to prove the following theorem

Theorem

IF f and g are continuous functions
THEN $h(x) = \max\{f(x), g(x)\}$ is also a continuous function.

You are allowed to use all results that we already know.
What is the fastest way to prove this?

Hint: There is a way to prove this quickly without writing any epsilons.

Computations!

Using that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, compute the following limits:

1. $\lim_{x\to 2} \frac{\sin x}{x}$
2. $\lim_{x\to 0} \frac{\sin(5x)}{x}$
3. $\lim_{x\to 0} \frac{\tan^2(2x^2)}{x^4}$
4. $\lim_{x\to 0} \frac{\sin e^x}{e^x}$
5. $\lim_{x\to 0} \frac{1-\cos x}{x}$
6. $\lim_{x\to 0} \frac{\tan^{10}(2x^{20})}{\sin^{200}(3x)}$

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7. $\lim_{x \to 0} \left[(\sin x) (\cos(2x)) (\tan(3x)) (\sec(4x)) (\csc(5x)) (\cot(6x)) \right]$

An extended version