MAT137

- Today: More indeterminate forms and L'Hôpital's Rule

- Videos for next Thursday's class: 6.13, 6.14.

Proving something is an indeterminate form

1. Prove that $\forall c \in \mathbb{R}$, $\exists a \in \mathbb{R}$ and functions f and g s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

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2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.
3. Prove that 1^∞, 0^0, and ∞^0 are indeterminate forms. (You will only get $c \geq 0$ this time)

Infinity minus infinity

Compute:

1. $\lim\limits_{x\to 0}\left[\dfrac{\csc x}{x}-\dfrac{\cot x}{x}\right]$

2. $\lim\limits_{x\to\infty}\left[\ln(x+2)-\ln(3x+4)\right]$

3. $\lim\limits_{x\to 1}\left[\dfrac{2}{x^2-1}-\dfrac{1}{x-1}\right]$

4. $\lim\limits_{x\to-\infty}\left[\sqrt{x^2+3x}-\sqrt{x^2-3x}\right]$

Exponential indeterminate forms

Compute:

1. $\lim\limits_{x\to 0} [1 + 2\sin(3x)]^{4\cot(5x)}$
2. $\lim\limits_{x\to\infty} \left(\dfrac{x+2}{x-2}\right)^{3x}$
3. $\lim\limits_{x\to 0^+} x^x$
4. $\lim\limits_{x\to \frac{\pi}{2}^-} (\tan x)^{\cos x}$
5. $\lim\limits_{x\to 0} \left(\dfrac{\sin x}{x}\right)^{\frac{1}{x^2}}$

Taylor preview

Let f be a function with domain $\mathbb{R}$.
Assume f is differentiable as many times as needed.

1. Find $a, b \in \mathbb{R}$ such that
$$\lim_{x \to 0} \frac{f(x) - [a + bx]}{x} = 0$$
2. Find $a, b, c \in \mathbb{R}$ such that
$$\lim_{x \to 0} \frac{f(x) - \left[a + bx + cx^2\right]}{x^2} = 0$$
3. Let $N \in \mathbb{N}$. Find a polynomial P_N such that
$$\lim_{x \to 0} \frac{f(x) - P_N(x)}{x^N} = 0$$

Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn't, then what is the value of such limit?

1. $\frac{0}{0}$
2. $\frac{0}{\infty}$
3. $\frac{0}{1}$
4. $\frac{\infty}{0}$
5. $\frac{\infty}{\infty}$
6. $\frac{1}{\infty}$
7. $0 \cdot \infty$
8. $\infty \cdot \infty$
9. $\sqrt{\infty}$
10. $\infty - \infty$
11. 1^{∞}
12. $1^{-\infty}$
13. 0^{0}
14. 0^{∞}
15. $0^{-\infty}$
16. ∞^{0}
17. ∞^{∞}
18. $\infty^{-\infty}$