Course website http://uoft.me/MAT137.

Test 2 is today! Good Luck.

Today: More indeterminate forms and L’Hôpital’s Rule

Videos for next Monday’s class: 6.11, 6.12, 6.13, 6.14, 6.15, 6.16
1. Prove that $\forall c \in \mathbb{R}, \exists a \in \mathbb{R}$ and functions $f$ and $g$ s.t.

$$\lim_{x \to a} f(x) = 0, \quad \lim_{x \to a} g(x) = 0, \quad \lim_{x \to a} \frac{f(x)}{g(x)} = c$$

This is how you show that $\frac{0}{0}$ is an indeterminate form.
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2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.
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2. Prove the same way that $\frac{\infty}{\infty}$, $0 \cdot \infty$, and $\infty - \infty$ are also indeterminate forms.

3. Prove that $1^{\infty}$, $0^{0}$, and $\infty^{0}$ are indeterminate forms. (You will only get $c \geq 0$ this time)
Infinity minus infinity

Compute:

1. \( \lim_{x \to 0} \left[ \frac{\csc x}{x} - \frac{\cot x}{x} \right] \)

2. \( \lim_{x \to \infty} [\ln(x + 2) - \ln(3x + 4)] \)

3. \( \lim_{x \to 1} \left[ \frac{2}{x^2 - 1} - \frac{1}{x - 1} \right] \)

4. \( \lim_{x \to -\infty} \left[ \sqrt{x^2 + 3x} - \sqrt{x^2 - 3x} \right] \)
Exponential indeterminate forms

Compute:

1. \[ \lim_{x \to 0} [1 + 2 \sin(3x)]^{4 \cot(5x)} \]

2. \[ \lim_{x \to \infty} \left( \frac{x + 2}{x - 2} \right)^{3x} \]

3. \[ \lim_{x \to 0^+} x^x \]

4. \[ \lim_{x \to \frac{\pi}{2}^-} (\tan x)^{\cos x} \]

5. \[ \lim_{x \to 0} \left( \frac{\sin x}{x} \right)^{1 \over x^2} \]
Let $f$ be a function with domain $\mathbb{R}$. Assume $f$ is differentiable as many times as needed.

1. Find $a, b \in \mathbb{R}$ such that
   \[
   \lim_{x \to 0} \frac{f(x) - [a + bx]}{x} = 0
   \]

2. Find $a, b, c \in \mathbb{R}$ such that
   \[
   \lim_{x \to 0} \frac{f(x) - [a + bx + cx^2]}{x^2} = 0
   \]

3. Let $N \in \mathbb{N}$. Find a polynomial $P_N$ such that
   \[
   \lim_{x \to 0} \frac{f(x) - P_N(x)}{x^N} = 0
   \]
Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn’t, then what is the value of such limit?

1. \( \frac{0}{0} \)
2. \( \frac{0}{\infty} \)
3. \( \frac{0}{1} \)
4. \( \frac{\infty}{0} \)
5. \( \frac{\infty}{\infty} \)
6. \( \frac{1}{\infty} \)
7. \( 0 \cdot \infty \)
8. \( \infty \cdot \infty \)
9. \( \sqrt{\infty} \)
10. \( \infty - \infty \)
11. \( 1^\infty \)
12. \( 1^{-\infty} \)
13. \( 0^0 \)
14. \( 0^\infty \)
15. \( 0^{-\infty} \)
16. \( \infty^0 \)
17. \( \infty^{-\infty} \)