• Today: Derivatives of exponentials and logarithms

• Homework before Thursday’s class: watch videos 4.12, 4.13, 4.14.
Warm up

Compute the derivative of the following functions:

1. \( f(x) = e^{\sin x + \cos x} \ln x \)
2. \( f(x) = \pi^{\tan x} \)
3. \( f(x) = \ln [e^x + \ln \ln \ln x] \)

Reminder: We know:

\[
\frac{d}{dx} e^x = e^x \\
\frac{d}{dx} a^x = a^x \ln a \\
\frac{d}{dx} \ln x = \frac{1}{x}
\]
A different type of logarithm

Calculate the derivative of

\[ f(x) = \log_{x+1}(x^2 + 1) \]

*Hint:* If you do not know where to start, remember the definition of logarithm:

\[ \log_a b = c \iff a^c = b. \]
Logarithmic differentiation

Calculate the derivative of

\[ g(x) = x^{\tan x}. \]
More logarithmic differentiation

Calculate the derivative of

\[ f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}. \]
More logarithmic differentiation

Calculate the derivative of

\[ f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}. \]

What is wrong with this answer?

\[
\begin{align*}
\ln f(x) &= (\cos x) \ln(\sin x) + (\sin x)(\ln \cos x) \\
\frac{d}{dx} \left[ \ln f(x) \right] &= \left( \frac{d}{dx} ((\cos x) \ln(\sin x)) \right) + \left( \frac{d}{dx} ((\sin x)(\ln \cos x)) \right) \\
\frac{f'(x)}{f(x)} &= - (\sin x) \ln(\sin x) + (\cos x) \frac{\cos x}{\sin x} \\
&\quad + (\cos x) \ln(\cos x) + (\sin x) \frac{-\sin x}{\cos x} \\
f'(x) &= f(x) \left[ - (\sin x) \ln(\sin x) + (\cos x) \ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x} \right]
\end{align*}
\]
Calculate the derivative of

\[ h(x) = \sqrt[3]{\frac{(\sin^6 x) \sqrt{x^7 + 6x + 2}}{3^x (x^{10} + 2x)^{10}}} \]