MAT137

- Today: Derivatives of exponentials and logarithms

- Homework before Thursday's class: watch videos 4.12, 4.13, 4.14.

Warm up

Compute the derivative of the following functions:

1. $f(x) = e^{\sin x + \cos x} \ln x$
2. $f(x) = \pi^{\tan x}$
3. $f(x) = \ln\left[e^x + \ln\ln\ln x\right]$

Reminder: We know:

- $\dfrac{d}{dx} e^x = e^x$
- $\dfrac{d}{dx} a^x = a^x \ln a$
- $\dfrac{d}{dx} \ln x = \dfrac{1}{x}$

A different type of logarithm

Calculate the derivative of

$$f(x) = \log_{x+1}(x^2 + 1)$$

Hint: If you do not know where to start, remember the definition of logarithm:

$$\log_a b = c \iff a^c = b.$$

Logarithmic differentiation

Calculate the derivative of

$$g(x) = x^{\tan x}.$$

More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

More logarithmic differentiation

Calculate the derivative of

$$f(x) = (\sin x)^{\cos x} + (\cos x)^{\sin x}.$$

What is wrong with this answer?

$$\begin{aligned}
\ln f(x) =& (\cos x)\ln(\sin x) + (\sin x)(\ln\cos x) \\
\frac{d}{dx}[\ln f(x)] =& \frac{d}{dx}[(\cos x)\ln(\sin x)] + \frac{d}{dx}[(\sin x)(\ln\cos x)] \\
\frac{f'(x)}{f(x)} =& -(\sin x)\ln(\sin x) + (\cos x)\frac{\cos x}{\sin x} \\
& + (\cos x)\ln(\cos x) + (\sin x)\frac{-\sin x}{\cos x}
\end{aligned}$$

$$f'(x) = f(x)\left[-(\sin x)\ln(\sin x) + (\cos x)\ln(\cos x) + \frac{\cos^2 x}{\sin x} - \frac{\sin^2 x}{\cos x}\right]$$

Hard derivatives made easier

Calculate the derivative of

$$h(x) = \sqrt[3]{\frac{\left(\sin^6 x\right)\sqrt{x^7 + 6x + 2}}{3^x \left(x^{10} + 2x\right)^{10}}}$$