Test 2 is on Friday, December 2

Today: Indeterminate forms and L’Hôpital’s Rule

Watch videos 6.10, 6.12, as well as 6.11 before next Wednesday’s class.
Calculate:

1. \( \lim_{x \to 2} \frac{x^2 + 2x - 6}{x^2 + 3x - 10} \)

2. \( \lim_{x \to 0} \frac{e^{2x^2} - \cos x}{x \sin x} \)

3. \( \lim_{x \to \infty} \frac{x^2}{e^x} \)

4. \( \lim_{x \to \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}} \)
Indeterminate?

Which of the following are indeterminate forms for limits? If any of them isn’t, then what is the value of such limit?

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More computations

Calculate:

1. \( \lim_{x \to \infty} (\sin x) \left( e^{1/x} - 1 \right) \)

2. \( \lim_{x \to \infty} x \sin \frac{2}{x} \)

3. \( \lim_{x \to \infty} x \cos \frac{2}{x} \)

4. \( \lim_{x \to 1} \left[ (\ln x) \tan \frac{\pi x}{2} \right] \)
Limits from graphs

Compute:

\[ \lim_{x \to 0} \frac{H(x)}{H(2 + 3x) - 1} \]
Sample solution of the last slide’s limit

Note that $H(x)$ and $H(2 + 3x) - 1$ are both differentiable as $x \to 0$. Moreover, $H(2 + 3x) - 1 \neq 0$ for values of $x$ near 0 but not equal to 0.

Also, $\lim_{x \to 0} H(x) = 0$ and $\lim_{x \to 0} (H(2 + 3x) - 1) = 0$. Therefore, we can apply L’Hôpital’s rule.

In order to do so, note that

- $H'(x) = 1$ as $x \to 0$,
- $(H(2 + 3x) - 1)' = H'(2 + 3x) \cdot 3$ as $x \to 0$, and
- $H'(x) = -2$ as $x \to 2$.

Therefore, $\lim_{x \to 0} H'(x) = 1$ and $\lim_{x \to 0} (H(2 + 3x) - 1)' = -6$. Thus

$$\lim_{x \to 0} \frac{H(x)}{H(2 + 3x) - 1} = \frac{-1}{6}$$