

- Test 2 is on Friday, December 2
- Today: Indeterminate forms and L'Hôpital's Rule
- Watch videos 6.10, 6.12, as well as 6.11 before next Wednesday's class.

Calculate:

$$1. \lim_{x \rightarrow 2} \frac{x^2 + 2x - 6}{x^2 + 3x - 10}$$

$$2. \lim_{x \rightarrow 0} \frac{e^{2x^2} - \cos x}{x \sin x}$$

$$3. \lim_{x \rightarrow \infty} \frac{x^2}{e^x}$$

$$4. \lim_{x \rightarrow \infty} \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

# Indeterminate?

Which of the following are indeterminate forms for limits?  
If any of them isn't, then what is the value of such limit?

1.  $\frac{0}{0}$

3.  $\frac{0}{1}$

5.  $\frac{\infty}{\infty}$

7.  $0 \cdot \infty$

2.  $\frac{0}{\infty}$

4.  $\sqrt{\infty}$

6.  $\frac{1}{\infty}$

8.  $\infty \cdot \infty$

Calculate:

$$1. \lim_{x \rightarrow \infty} (\sin x) \left( e^{1/x} - 1 \right)$$

$$2. \lim_{x \rightarrow \infty} x \sin \frac{2}{x}$$

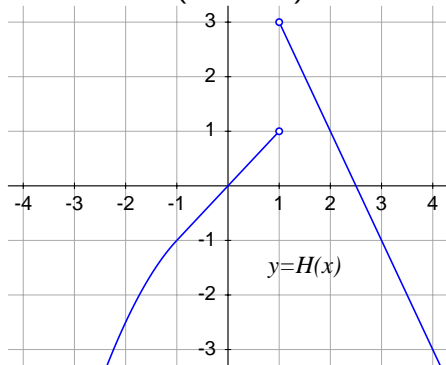
$$3. \lim_{x \rightarrow \infty} x \cos \frac{2}{x}$$

$$4. \lim_{x \rightarrow 1} \left[ (\ln x) \tan \frac{\pi x}{2} \right]$$

# Limits from graphs

Compute:

$$\lim_{x \rightarrow 0} \frac{H(x)}{H(2 + 3x) - 1}$$



## Sample solution of the last slide's limit

Note that  $H(x)$  and  $H(2 + 3x) - 1$  are both differentiable as  $x \rightarrow 0$ . Moreover,  $H(2 + 3x) - 1 \neq 0$  for values of  $x$  near 0 but not equal to 0.

Also,  $\lim_{x \rightarrow 0} H(x) = 0$  and  $\lim_{x \rightarrow 0} (H(2 + 3x) - 1) = 0$ . Therefore, we can apply L'Hôpital's rule.

In order to do so, note that

- $H'(x) = 1$  as  $x \rightarrow 0$ ,
- $(H(2 + 3x) - 1)' = H'(2 + 3x) \cdot 3$  as  $x \rightarrow 0$ , and
- $H'(x) = -2$  as  $x \rightarrow 2$ .

Therefore,  $\lim_{x \rightarrow 0} H'(x) = 1$  and  $\lim_{x \rightarrow 0} (H(2 + 3x) - 1)' = -6$ . Thus

$$\lim_{x \rightarrow 0} \frac{H(x)}{H(2 + 3x) - 1} = -\frac{1}{6}$$