- Today: One-to-one functions and inverse functions.

- Homework before Wednesday's class:
  watch videos 4.5, 4.7, 4.8, 4.9, as well as 4.6, 4.10, 4.11 (they are still short!).

From the last lecture: One-to-one functions

1. Write the definition of a one-to-one function.

2. Let f be a function defined by $f(x) = 2x^3 + 7$.
   Prove that f is one-to-one.

3. Give the formula for the inverse function $x = f^{-1}(y)$.

Derivative of the inverse function

1. Let f be a function defined by $f(x) = 2x^3 + 7$.
   What is the value $f^{-1}(-9)$?

2. Find the derivative $(f^{-1})'(y)$ at $y = -9$ by using the formula for the derivative of the inverse function for f.

3. Compare this with the derivative of the explicit form for $x = f^{-1}(y)$ at $y = -9$.

Composition of one-to-one functions

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$. Prove the following theorem.

Theorem

Let f and g be functions.
IF f and g are one-to-one,
THEN $f \circ g$ is one-to-one.

Suggestion:

1. Write the definition of what you want to prove.
2. Figure out the formal structure of the proof.
3. Complete the proof (use the hypotheses!)

Composition of one-to-one functions – 2

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Is the following claim TRUE or FALSE? Prove it or give a counterexample.

Claim

Let f and g be functions.
IF $f \circ g$ is one-to-one,
THEN f and g are one-to-one.

Assume for simplicity that all functions in this problem have domain $\mathbb{R}$.

Let f and g be functions. Assume they each have an inverse.

Is $(f \circ g)^{-1} = f^{-1} \circ g^{-1}$?

- If YES, prove it.
- If NO, fix the statement.

If you do not know how to start, experiment with the functions

$$f(x) = x + 1, \qquad g(x) = 2x.$$

Draw a graph from properties

Sketch the graph of a function g satisfying all the following properties:

1. The domain of g is $\mathbb{R}$.
2. g is continuous everywhere except at -2.
3. g is differentiable everywhere except at -2 and 1.
4. g has an inverse function.
5. $g(0) = 2$
6. $g'(0) = 2$
7. $\left(g^{-1}\right)'(-3) = -2$.