Test 2 is on Friday, December 3.

Today: the Mean Value Theorem (MVT).

Homework before Tuesday’s class: watch videos 5.10, 5.11, as well as 5.12.
Positive derivative implies increasing

Use the MVT to prove

**Theorem**

Let $a < b$. Let $f$ be a differentiable function on $(a, b)$. 
- IF $\forall x \in (a, b), f'(x) > 0$,
- THEN $f$ is increasing on $(a, b)$. 
Positive derivative implies increasing

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1. Recall the definition of what you are trying to prove.
2. From that definition, figure out the structure of the proof.
3. If you have used a theorem, did you verify the hypotheses?
4. Are there words in your proof, or just equations?
Theorem

Let $a < b$. Let $f$ be a differentiable function on $(a, b)$.

- IF $\forall x \in (a, b), f'(x) > 0$,
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Proof.

- From the MVT, $f'(c) = \frac{f(b) - f(a)}{b - a}$
- We know $b - a > 0$ and $f'(c) > 0$
- Therefore $f(b) - f(a) > 0$, so $f(b) > f(a)$
- $f$ is increasing.
Cauchy’s MVT - Part 1

Here is a new theorem:

We want to prove this Theorem

Let $a < b$. Let $f$ and $g$ be functions defined on $[a, b]$. IF (some conditions) THEN $\exists c \in (a, b)$ such that $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$.

What is wrong with this “proof”? By MVT, $\exists c \in (a, b)$ s.t. $f'(c) = f(b) - f(a)$ and $b - a$. By MVT, $\exists c \in (a, b)$ s.t. $g'(c) = g(b) - g(a)$ and $b - a$. Divide the two equations and we get what we wanted.
Cauchy’s MVT - Part 1

Here is a new theorem:

We want to prove this Theorem

Let \( a < b \). Let \( f \) and \( g \) be functions defined on \([a, b]\).

IF (some conditions)

THEN \( \exists c \in (a, b) \) such that

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\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}
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What is wrong with this “proof”?

- By MVT, \( \exists c \in (a, b) \) s.t. \( f'(c) = \frac{f(b) - f(a)}{b - a} \)
- By MVT, \( \exists c \in (a, b) \) s.t. \( g'(c) = \frac{g(b) - g(a)}{b - a} \)
- Divide the two equations and we get what we wanted.
We want to prove this theorem

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We want to prove this theorem

Let $a < b$. Let $f$ and $g$ be functions defined on $[a, b]$. IF (some conditions) THEN $\exists c \in (a, b)$ such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

1. There is one number $M \in \mathbb{R}$ so that you will be able to apply Rolle’s Theorem to the new function $H(x) = f(x) - Mg(x)$ on the interval $[a, b]$. What is $M$?
We want to prove this theorem

Let \( a < b \). Let \( f \) and \( g \) be functions defined on \([a, b]\).
IF (some conditions)
THEN \( \exists c \in (a, b) \) such that
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\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}
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1. There is one number \( M \in \mathbb{R} \) so that you will be able to apply Rolle’s Theorem to the new function \( H(x) = f(x) - Mg(x) \) on the interval \([a, b]\). What is \( M \)?
2. Apply Rolle’s Theorem to \( H \). What do you conclude?
3. Fill in the missing hypotheses in the theorem above.
4. Prove it.
Proving difficult identities

Prove that, for every $x \geq 0$,

$$\arcsin \frac{1-x}{1+x} + 2 \arctan \sqrt{x} = \frac{\pi}{2}$$

*Hint:* Take derivatives.