MAT137

- Today: Properties of series.

- Homework before Thursday's class: watch videos 13.8, 13.9.

Rapid questions: More improper integrals

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{x}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$

Rapid questions: More improper integrals

1. $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx$
2. $\displaystyle\int_1^\infty \frac{1}{x}\,dx$
3. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
4. $\displaystyle\int_1^\infty \frac{x+1}{x^3+2}\,dx$
5. $\displaystyle\int_1^\infty \frac{\sqrt{x^2+5}}{x^3+6}\,dx$
6. $\displaystyle\int_1^\infty \frac{x^2+3}{\sqrt{x^5+2}}\,dx$

Geometric series

Calculate the value of the following series:

1. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\dots$
2. $\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\dots$
3. $\frac{3}{2}-\frac{9}{4}+\frac{27}{8}-\frac{81}{16}+\dots$
4. $1+\frac{1}{2^{0.5}}+\frac{1}{2}+\frac{1}{2^{1.5}}+\frac{1}{2^2}+\frac{1}{2^{2.5}}+\dots$
5. $\sum_{n=1}^{\infty}(-1)^n\frac{3^n}{2^{2n+1}}$
6. $\sum_{n=k}^{\infty}x^n$

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$
2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$
3. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.
4. IF the series $\sum_{n=0}^{\infty} a_n$ converges

   THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.

True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

6. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing, THEN $\forall n \geq 0$, $a_n > 0$.

7. IF $\lim_{n\to\infty} a_n = 0$, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

8. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{n\to\infty} a_n = 0$.

Is $0.999999\cdots = 1$?

Is $0.999999\cdots = 1$?

1. Write the number $0.9999999\cdots$ as a series
   Hint: $427 = 400 + 20 + 7$.

2. Compute the first few partial sums

3. Add up the series.
   Hint: it is geometric.

Series expansion

From the geometric series, we know that when $|x| < 1$, we can expand the function $f(x) = \dfrac{1}{1-x}$ as a series:

$$f(x) = \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

Find similar ways to write the following functions as series:

1. $g(x) = \dfrac{1}{1+x}$ 2. $h(x) = \dfrac{1}{1-x^2}$ 3. $k(x) = \dfrac{1}{2-x}$