Today: Definition of series.

Homework before Wednesday’s class: watch videos 13.5, 13.6, 13.7.
Rapid questions: review of improper integrals

1. \[ \int_{1}^{\infty} \frac{1}{x^2} \, dx \]

2. \[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]

3. \[ \int_{1}^{\infty} \frac{1}{x^2 + \sqrt{x}} \, dx \]
Rapid questions: review of improper integrals

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3. \[ \int_{1}^{\infty} \frac{1}{x^2 + \sqrt{x}} \, dx \]

4. \[ \int_{0}^{1} \frac{1}{x^2} \, dx \]

5. \[ \int_{0}^{1} \frac{1}{\sqrt{x}} \, dx \]

6. \[ \int_{0}^{1} \frac{1}{x^2 + \sqrt{x}} \, dx \]
Rapid questions: review of improper integrals

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7. \( \int_{0}^{\infty} \frac{1}{x^2} \, dx \)

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9. \( \int_{0}^{\infty} \frac{1}{x^2 + \sqrt{x}} \, dx \)
Trig series: convergent or divergent?

1. $\sum_{n=0}^{\infty} \sin(n\pi)$

2. $\sum_{n=0}^{\infty} \cos(n\pi)$
A telescopic series

I want to calculate the value of the series \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \).

1. Find a formula for the \( k \)-th partial sum \( S_k = \sum_{n=1}^{k} \frac{1}{n^2 + 2n} \).

   \text{Hint: Write } \frac{1}{n^2 + n} = \frac{A}{n} + \frac{B}{n + 2}

2. Using the definition of series, compute the value of

   \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \)
What is wrong with this calculation? Fix it

Claim:

\[ \sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2 \]

"Justification"

\[ \sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \]

\[ = \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1) \]

\[ = (\ln 2 + \ln 3 + \ln 4 + \ldots) - (\ln 3 + \ln 4 + \ldots) \]

\[ = \ln 2 \]
In the next problem set I want to give you a series and ask you to calculate its value from the definition. I want the sequence of partial sums $\{S_n\}_{n=1}^{\infty}$ to be

$$\forall n \geq 1, \quad S_n = \frac{1}{n^2}.$$ 

What series should I ask you to calculate?
Harmonic series

For each $n > 0$ we define

$$r_n = \text{smallest power of 2 that is greater than or equal to } n$$

Consider the series $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$

1. Compute $r_1$ through $r_8$
2. Compute the partial sums $S_1, S_2, S_4, S_8$ for the series $S$.
3. Calculate $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.
4. “Compare” the harmonic series $H = \sum_{n=1}^{\infty} \frac{1}{n}$ and $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.

Is $H$ convergent or divergent?