- Today: Definition of series.

- Homework before Wednesday's class: watch videos 13.5, 13.6, 13.7.

Rapid questions: review of improper integrals

1. $\int_1^\infty \frac{1}{x^2}\,dx$
2. $\int_1^\infty \frac{1}{\sqrt{x}}\,dx$
3. $\int_1^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

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Rapid questions: review of improper integrals

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7. $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$
8. $\displaystyle\int_0^\infty \frac{1}{\sqrt{x}}\,dx$
9. $\displaystyle\int_0^\infty \frac{1}{x^2+\sqrt{x}}\,dx$

Trig series: convergent or divergent?

1. $\sum_{n=0}^{\infty} \sin(n\pi)$

2. $\sum_{n=0}^{\infty} \cos(n\pi)$

A telescopic series

I want to calculate the value of the series $\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$.

1. Find a formula for the k-th partial sum $S_k = \sum_{n=1}^{k} \frac{1}{n^2+2n}$.

   Hint: Write $\frac{1}{n^2+n} = \frac{A}{n} + \frac{B}{n+2}$

2. Using the definition of series, compute the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$$

What is wrong with this calculation? Fix it

Claim:

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

"Justification"

$$\begin{aligned}
\sum_{n=2}^{\infty} \ln \frac{n}{n+1} &= \sum_{n=2}^{\infty} [\ln n - \ln(n+1)] \\
&= \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1) \\
&= (\ln 2 + \ln 3 + \ln 4 + \ldots) - (\ln 3 + \ln 4 + \ldots) \\
&= \ln 2
\end{aligned}$$

In the next problem set I want to give you a series and ask you to calculate its value from the definition. I want the sequence of partial sums $\{S_n\}_{n=1}^{\infty}$ to be

$$\forall n \geq 1,\ S_n = \frac{1}{n^2}.$$

What series should I ask you to calculate?

Harmonic series

For each $n > 0$ we define

$$r_n = \text{ smallest power of 2 that is greater than or equal to } n$$

Consider the series $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$

1. Compute r_1 through r_8
2. Compute the partial sums S_1, S_2, S_4, S_8 for the series S.
3. Calculate $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.
4. "Compare" the harmonic series $H = \sum_{n=1}^{\infty} \frac{1}{n}$ and $S = \sum_{n=1}^{\infty} \frac{1}{r_n}$.

   Is H convergent or divergent?