Today: Properties of series.

Homework before Friday’s class: watch videos 13.8, 13.9.
Rapid questions: More improper integrals

1. \[ \int_{1}^{\infty} \frac{1}{x^2} \, dx \]

2. \[ \int_{1}^{\infty} \frac{1}{x} \, dx \]

3. \[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]
Rapid questions: More improper integrals

1. \[ \int_{1}^{\infty} \frac{1}{x^2} \, dx \]

2. \[ \int_{1}^{\infty} \frac{1}{x} \, dx \]

3. \[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]

4. \[ \int_{1}^{\infty} \frac{x + 1}{x^3 + 2} \, dx \]

5. \[ \int_{1}^{\infty} \frac{\sqrt{x^2 + 5}}{x^3 + 6} \, dx \]

6. \[ \int_{1}^{\infty} \frac{x^2 + 3}{\sqrt{x^5 + 2}} \, dx \]
Geometric series

Calculate the value of the following series:

1. \(1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \ldots\)

2. \(\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32} - \ldots\)

3. \(\frac{3}{2} - \frac{9}{4} + \frac{27}{8} - \frac{81}{16} + \ldots\)

4. \(1 + \frac{1}{2^{0.5}} + \frac{1}{2} + \frac{1}{2^{1.5}} + \frac{1}{2^2} + \frac{1}{2^{2.5}} + \ldots\)

5. \(\sum_{n=1}^{\infty} (-1)^n \frac{3^n}{2^{2n+1}}\)

6. \(\sum_{n=k}^{\infty} x^n\)
True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

1. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $a_n > 100$

2. IF the series $\sum_{n=0}^{\infty} a_n$ is divergent, THEN $\exists n \in \mathbb{N}$ such that $S_n > 100$

3. IF the series $\sum_{n=0}^{\infty} a_n$ converges

THEN the series $\sum_{n=100}^{\infty} a_n$ converges to a smaller number.

4. IF the series $\sum_{n=0}^{\infty} a_n$ converges

THEN the sequence $\{S_n\}_{n=0}^{\infty}$ is eventually monotonic.
True or False – Series

Let $\sum_{n=0}^{\infty} a_n$ be a series. Let $\{S_n\}_{n=0}^{\infty}$ be its partial-sum sequence.

5. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is bounded and eventually monotonic, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

6. IF the sequence $\{S_n\}_{n=0}^{\infty}$ is increasing, THEN $\forall n \geq 0$, $a_n > 0$.

7. IF $\lim_{n \to \infty} a_n = 0$, THEN the series $\sum_{n=0}^{\infty} a_n$ is convergent.

8. IF the series $\sum_{n=0}^{\infty} a_n$ is convergent, THEN $\lim_{n \to \infty} a_n = 0$. 
Is $0.999999 \cdots = 1$?

1. Write the number $0.9999999 \cdots$ as a series.
2. Compute the first few partial sums.
3. Add up the series.

Hint: it is geometric.
Is $0.999999 \cdots = 1$?

1. Write the number $0.999999 \cdots$ as a series
   
   *Hint*: $427 = 400 + 20 + 7$.

2. Compute the first few partial sums

3. Add up the series.
   
   *Hint*: it is geometric.
From the geometric series, we know that when $|x| < 1$, we can expand the function $f(x) = \frac{1}{1 - x}$ as a series:

$$f(x) = \frac{1}{1 - x} = \sum_{n=0}^{\infty} x^n$$

Find similar ways to write the following functions as series:

1. $g(x) = \frac{1}{1 + x}$
2. $h(x) = \frac{1}{1 - x^2}$
3. $k(x) = \frac{1}{2 - x}$