Today: Definition of series.

Homework before Wednesday’s class: watch videos 13.5, 13.6, 13.7.
Rapid questions: review of improper integrals

1. \[ \int_{1}^{\infty} \frac{1}{x^2} \, dx \]

2. \[ \int_{1}^{\infty} \frac{1}{\sqrt{x}} \, dx \]

3. \[ \int_{1}^{\infty} \frac{1}{x^2 + \sqrt{x}} \, dx \]
Rapid questions: review of improper integrals

1. \( \int_1^\infty \frac{1}{x^2} \, dx \)

2. \( \int_1^\infty \frac{1}{\sqrt{x}} \, dx \)

3. \( \int_1^\infty \frac{1}{x^2 + \sqrt{x}} \, dx \)

4. \( \int_0^1 \frac{1}{x^2} \, dx \)

5. \( \int_0^1 \frac{1}{\sqrt{x}} \, dx \)

6. \( \int_0^1 \frac{1}{x^2 + \sqrt{x}} \, dx \)
Rapid questions: review of improper integrals

1. $\int_1^\infty \frac{1}{x^2} \, dx$
2. $\int_1^\infty \frac{1}{\sqrt{x}} \, dx$
3. $\int_1^\infty \frac{1}{x^2 + \sqrt{x}} \, dx$
4. $\int_0^1 \frac{1}{x^2} \, dx$
5. $\int_0^1 \frac{1}{\sqrt{x}} \, dx$
6. $\int_0^1 \frac{1}{x^2 + \sqrt{x}} \, dx$
7. $\int_0^\infty \frac{1}{x^2} \, dx$
8. $\int_0^\infty \frac{1}{\sqrt{x}} \, dx$
9. $\int_0^\infty \frac{1}{x^2 + \sqrt{x}} \, dx$
Trig series: convergent or divergent?

1. \[ \sum_{n=0}^{\infty} \sin(n\pi) \]

2. \[ \sum_{n=0}^{\infty} \cos(n\pi) \]
A telescopic series

I want to calculate the value of the series \( \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \).

1. Find a formula for the \( k \)-th partial sum \( S_k = \sum_{n=1}^{k} \frac{1}{n^2 + 2n} \).

Hint: Write \( \frac{1}{n^2 + n} = \frac{A}{n} + \frac{B}{n + 2} \)

2. Using the definition of series, compute the value of

\[ \sum_{n=1}^{\infty} \frac{1}{n^2 + 2n} \]
What is wrong with this calculation? Fix it

Claim:

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \ln 2$$

"Justification"

$$\sum_{n=2}^{\infty} \ln \frac{n}{n+1} = \sum_{n=2}^{\infty} [\ln n - \ln(n+1)]$$

$$= \sum_{n=2}^{\infty} \ln(n) - \sum_{n=2}^{\infty} \ln(n+1)$$

$$= (\ln 2 + \ln 3 + \ln 4 + \ldots) - (\ln 3 + \ln 4 + \ldots)$$

$$= \ln 2$$
In the next problem set I want to give you a series and ask you to calculate its value from the definition. I want the sequence of partial sums \( \{S_n\}_{n=1}^{\infty} \) to be

\[
\forall n \geq 1, \quad S_n = \frac{1}{n^2}.
\]

What series should I ask you to calculate?
Harmonic series

For each \( n > 0 \) we define

\[
    r_n = \text{smallest power of 2 that is greater than or equal to } n
\]

Consider the series \( S = \sum_{n=1}^{\infty} \frac{1}{r_n} \)

1. Compute \( r_1 \) through \( r_8 \)
2. Compute the partial sums \( S_1, S_2, S_4, S_8 \) for the series \( S \).
3. Calculate \( S = \sum_{n=1}^{\infty} \frac{1}{r_n} \).

4. “Compare” the harmonic series \( H = \sum_{n=1}^{\infty} \frac{1}{n} \) and \( S = \sum_{n=1}^{\infty} \frac{1}{r_n} \).

Is \( H \) convergent or divergent?