- Today: Constructing new power series.

- Homework before Tuesday's class: watch videos 14.12, 14.14.

Write the following functions as power series centered at 0. Write them first with sigma notation, and then write out the first few terms.

1. $f(x) = \dfrac{x^2}{1+x}$
2. $f(x) = (e^x)^2$
3. $f(x) = \sin\left(2x^3\right)$
4. $f(x) = \cos^2 x$
5. $f(x) = \ln \dfrac{1+x}{1-x}$
6. $f(x) = \dfrac{1}{(1+x^2)(1+x)}$

Note: You do not need to take any derivatives. You can reduce them all to other Maclaurin series you know.

Arctan

1. Write the function

$$f(x) = \arctan x$$

as a power series centered at 0.
Hint: Compute the first derivative. Then stop to think.

Arctan

1. Write the function

$$f(x) = \arctan x$$

as a power series centered at 0.
Hint: Compute the first derivative. Then stop to think.

2. What is $f^{(203)}(0)$?

Obtain the **first four non-zero terms** of the Maclaurin series of these functions:

1. $f(x) = e^x \sin x$

2. $g(x) = e^{\sin x}$

Hint: Treat the power series the same way you would treat a polynomial.

Other operations with Taylor series

Obtain the **first four non-zero terms** of the Maclaurin series of these functions:

1. $f(x) = e^x \sin x$

2. $g(x) = e^{\sin x}$

Hint: Treat the power series the same way you would treat a polynomial.

Follow-up question: What is $g^{(4)}(0)$?

Parity

1. Let f be an odd, C^∞ function. What can you say about its Maclaurin series? What if f is even?

   Hint: Think of sin and cos.

2. Prove it.

   Hint: Use the general formula for the Maclaurin series. What can you say about $h(0)$ if h is odd? If h is even?

Tangent

There is no nice, compact formula for the Maclaurin series of tan, but we can obtain the first few terms. Set

$$\tan x = c_1 x + c_3 x^3 + c_5 x^5 + \dots$$

By definition of tan, we have:

$$\sin x = (\cos x)(\tan x)$$

So

$$\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots\right] = \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots\right] \cdot \left[c_1 x + c_3 x^3 + c_5 x^5 + \dots\right]$$

Expand. Obtain equations for the coefficients c_n and solve for the first few ones.