Today: New power series.

Write the following functions as power series centered at 0. Write them first with sigma notation, and then write out the first few terms.

1. \( f(x) = \frac{x^2}{1 + x} \)
2. \( f(x) = (e^x)^2 \)
3. \( f(x) = \sin(2x^3) \)
4. \( f(x) = \cos^2 x \)
5. \( f(x) = \ln \frac{1 + x}{1 - x} \)
6. \( f(x) = \frac{1}{(1 + x^2)(1 + x)} \)

**Note:** You do not need to take any derivatives. You can reduce them all to other Maclaurin series you know.
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\[ f(x) = \arctan x \]

as a power series centered at 0.

*Hint:* Compute the first derivative. Then stop to think.
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2. What is \( f^{(203)}(0) \)?
Other operations with Taylor series

Obtain the **first four non-zero terms** of the Maclaurin series of these functions:

1. \( f(x) = e^x \sin x \)

2. \( g(x) = e^{\sin x} \)

*Hint*: Treat the power series the same way you would treat a polynomial.
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Follow-up question: What is \( g^{(4)}(0) \)?
1. Let \( f \) be an odd, \( C^\infty \) function. What can you say about its Maclaurin series? What if \( f \) is even?

\textit{Hint:} Think of \( \sin \) and \( \cos \).

2. Prove it.

\textit{Hint:} Use the general formula for the Maclaurin series. What can you say about \( h(0) \) if \( h \) is odd? If \( h \) is even?
There is no nice, compact formula for the Maclaurin series of tan, but we can obtain the first few terms. Set

$$\tan x = c_1 x + c_3 x^3 + c_5 x^5 + \ldots$$

By definition of tan, we have:

$$\sin x = (\cos x)(\tan x)$$

So

$$\left[x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\right] = \left[1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots\right] \cdot \left[c_1 x + c_3 x^3 + c_5 x^5 + \ldots\right]$$

Expand. Obtain equations for the coefficients $c_n$ and solve for the first few ones.