Today: Taylor series.

Homework before Wednesday’s class: watch videos 14.7, 14.8.
1. Let \( f(x) = x^3 \)
Write the 2nd Taylor polynomial \( P_2 \) for \( f \) at 0
Verify that it is the correct answer using each of the 3 definitions.
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   Write the 2nd Taylor polynomial $P_2$ for $f$ at 0
   Verify that it is the correct answer using each of the 3 definitions.

2. Write the 2nd Taylor polynomial for $f$ at 1.

3. Write the 3rd Taylor polynomial for $f$ at 1
Interval of convergence

You have learned the Maclaurin series for the following functions

\[ f(x) = e^x, \quad g(x) = \sin x, \quad h(x) = \cos x \]

Compute the interval of convergence of each of these three series.
Taylor series not at 0

Write the Taylor series...

1. for $f(x) = e^x$ at $a = 2$
2. for $g(x) = \sin x$ at $a = \frac{\pi}{4}$
3. for $H(x) = \frac{1}{x}$ at $a = 3$

You can do these problems in two ways:

- Method 1: Compute the first few derivatives, guess the pattern (and prove it by induction).
- Method 2: Use the substitution $u = x - a$ and reduce it to an old problem (without computing any derivative).
Write the following functions as power series centered at 0:

1. $g(x) = \ln(1 + x)$
2. $h(x) = \arctan x$
3. $f(x) = \sin^2 x$.

_Hint:_ For each of the first two, take one single derivative. Then stop to think.