- Today: Taylor polynomials.
- Homework before Tuesday's class: watch videos 14.5, 14.6.

The definitions of Taylor polynomial

Let f be a function defined at and near $a \in \mathbb{R}$. Let $n \in \mathbb{N}$.
Let P_n be the n-th Taylor polynomial for f at a.
Which ones of these is true?

1. P_n is an approximation for f of order n near a.
2. f is an approximation for P_n of order n near a.
3. $\lim_{x \to a} [f(x) - P_n(x)] = 0$
4. $\lim_{x \to a} \frac{f(x) - P_n(x)}{(x-a)^n} = 0$
5. $\exists$ a function R_n s.t. $f(x) = P_n(x) + R_n(x)$ and $\lim_{x \to a} \frac{R_n(x)}{(x-a)^n} = 0$
6. $f^{(n)}(a) = P_n^{(n)}(a)$
7. $\forall k = 0, 1, 2, \ldots, n, \quad f^{(k)}(a) = P_n^{(k)}(a)$

Approximating functions

Which one of the following functions is a better approximation for $F(x) = e^x$ near 0?

1. $f(x) = 1 + x + \frac{x^2}{2}$
2. $g(x) = \sin x + \cos x + x^2$
3. $h(x) = e^{-x} + 2x$

[graph]

An explicit equation for Taylor polynomials

1. Find a polynomial P of degree 3 that satisfies

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

2. Find *all* polynomials P of degree ≤ 6 that satisfy

$$P(0) = 1, \quad P'(0) = 5, \quad P''(0) = 3, \quad P'''(0) = -7$$

3. Find a polynomial P of smallest possible degree that satisfies

$$P(0) = A, \quad P'(0) = B, \quad P''(0) = C, \quad P'''(0) = D$$

4. Find an explicit formula for the 3-rd Taylor polynomial for a function f at 0.
5. Find an explicit formula for the n-th Taylor polynomial for a function f at 0.

Examples

Use your shiny new explicit formula to compute the Taylor polynomials of degree ≤ 6 for these functions at 0:

1. $f(x) = e^x$
2. $g(x) = \sin x$
3. $h(x) = \cos x$

Curvature

Definition

Let C be a curve and let P be a point in C.

- The *osculating circle* of C at P is the circle that is the best approximation for C near P.
- The *curvature* of C at P is $\frac{1}{R}$, where R is the radius of the osculating circle.

Curvature

Definition

Let C be a curve and let P be a point in C.

- The *osculating circle* of C at P is the circle that is the best approximation for C near P.
- The *curvature* of C at P is $\frac{1}{R}$, where R is the radius of the osculating circle.

Let $a > 0$. Let C be the graph of $y = ax^2$. Find the radius of the osculating circle of C at $(0, 0)$.

Curvature

Definition

Let C be a curve and let P be a point in C.

- The *osculating circle* of C at P is the circle that is the best approximation for C near P.
- The *curvature* of C at P is $\frac{1}{R}$, where R is the radius of the osculating circle.

Let $a > 0$. Let C be the graph of $y = ax^2$. Find the radius of the osculating circle of C at $(0,0)$. You can try two different things:

1. Go to `http://tinyurl.com/osculatingcircle` and play with the sliders. Make a conjecture.
2. Write the equation of a circle tangent to C at $(0,0)$ with radius R. Then use the definition of "good approximation" to find the value of R.